BEHAVIOR AND STABILITY
OF A CAR FOLLOWING MODEL

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BEHAVIOR AND STABILITY

OF A CAR FOLLOWING MODEL

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SUMMARY

Car following models deal with the interactions within a chain of cars and relationships between concentration and velocity. Many mathematical models have been proposed and have contributed to an understanding of traffic flow and the interactions between cars. However, these models are not adequate to analyze the influence and interactions of such important factors as the limits of car performance, the effect of desired headway, and the effect of brake lights.

Thus a car following simulation model, which can deal with the above mentioned factors, is developed based on previous research, experiments and data collection. This model is written in DYNAMO, and it is flexible with respect to parameter changes and some structural changes. This model has been applied to such situations as emergency stop, stop-run-stop maneuvers, merging, and flow in a bottleneck. There are also examined such phenomena as collisions, wave propagation, and wave amplitude.

CHAPTER I

INTRODUCTION

Purpose

The importance of understanding vehicular traffic flow has been increasing, and much research has been done on the subject. Car following models represent one of the more basic types of research in this area. These models deal with the interactions within a chain of cars and relationships between concentration and velocity. Many theoretical mathematical models have been proposed and some of the simpler ones have been validated by experiments. These mathematical models have contributed to an understanding of traffic flow and the interactions between cars. However, these models are not adequate for analyzing the influence and interactions of such various factors as reaction time, acceleration limit, and driver's response sensitivity, because it is very difficult for such models to deal with performance limits, the effect of the brake light, the desired following distance, etc.

The purpose of this thesis is to develop a car following simulation model which can deal with the above-mentioned factors, to apply that model to some specific situations, and to analyze these situations.

Method of Approach

Previously developed car following theories, models and experiments which were available were investigated for possible use in building the car following simulation model, and such data as the driver's reaction time, the acceleration limit, and maximum braking force were collected.

After examination of the literature, Greenberg's mathematical model (1) was chosen as the basic model to be developed. DYNAMO (2) was applied as the simulation language, and the pipeline delay was selected to represent the reaction time.

By means of the above-mentioned investigation and data collection and after modification through several trial runs, a car following simulation which possesses the following characteristics was completed:

(1) It consists of a string of ten cars in a single lane.

(2) It is assumed that each driver responds only to the car in front of him.

(3) The effects of brake lights, limits of capacity of cars, and acceleration noise are considered.

The developed car following model has been applied to the cases of Emergency Stop, Stop-Run-Stop Maneuver, Merging from a Ramp, and Flow in a Bottleneck. Analysis has been done mainly by using fractional experimental designs.

CHAPTER II

CAR FOLLOWING MODELS

Mathematical Equations of Car Following Models

Car following theory applies to single-lane traffic with no overtaking. The theory is based on the assumption that each driver reacts in some specific fashion to a stimulus from the vehicle or vehicles ahead of him. In general, acceleration noise which is the variation in acceleration without the driver's intention, is ignored in this theory. Car following theory attempts to describe mathematically the way vehicles move on a road and to determine qualitatively what happens to the dynamics of this chain when there is a fluctuation in the motion.

The basic equation of car following theory is:

RESPONSE = (SENSITIVITY) x (STIMULUS)

To translate this equation into mathematical terms, quantitative values must be assigned to each factor. It has been generally recognized that response is most accurately defined as the acceleration of the vehicle, since this is directly controlled by the driver. Experiments by Chandler et al. (3) have shown that there is a high correlation between the response of a driver and the relative speed of his vehicle and the one ahead: thus, the stimulus is taken as this relative speed. Therefore, if we denote the sensitivity by λ, the car following equation becomes:

$$\ddot{X}_{n+1}(t + T) = \lambda\left(\dot{X}_n(t) - \dot{X}_{n+1}(t)\right) \tag{2-2}$$

where $X_n(t)$ = position of the nth vehicle at time t

$\dot{X}_n(t)$ = velocity of nth vehicle at time t

$\ddot{X}_n(t)$ = acceleration of nth vehicle at time t

T = reaction time (time lag of response to stimulus)

The variations in several car following theories are principally in the value taken for sensitivity, λ (see Appendix A for units):

(i) $\lambda = c_1$; constant

(ii) $\lambda = c_2/s$; the reciprocal of spacing

(iii) $\lambda = c_3/s^2$; the reciprocal of spacing squared

(iv) $\lambda = c_4\dot{X}_{n+1}/s^2$; the velocity of the following vehicle and the reciprocal of spacing squared.

(v) $\lambda = \begin{cases} a & s \leq Sc \\ b & s > Sc \end{cases}$

where Sc is a critical spacing. This is a step function.

The constant sensitivity (i) is the simplest one and it is assumed that the driver behaves in proportion to the relative speed regardless of his headway. Then this relation will be kept within a some narrow range of headway. In the second and the third equations it is assumed that the driver responds strongly to the relative speed if his headway is small. These equations seem to be superior to the constant sensitivity with respect to considering the headway. Experiments by Gazis et al. (4) show the superiority of the second equation (ii) over the first one. In addition to the consideration to headway, the fourth equation includes the factor of the current speed of the following car. The reason for this factor is that at higher speeds, the driver might be required to respond more rapidly. This seems to follow from common sense, but there does not seem to be any experimental data that supports this equation from a microscopic aspect, i.e. car following behavior itself. On the other hand, Edie's work (5) supports this equation from the macroscopic point of view. The last, fifth equation seems to suggest that at greater traffic densities the response time becomes smaller.

All of the above expressions can be included in the general expression:

$$\lambda = \frac{c\dot{X}_{n+1}{}^{m}(t + T)}{\left[X_n(t) - X_{n+1}(t)\right]^{\ell}} \tag{2-3}$$

where l and m are constants.

The Models and Steady State

The two important characteristics of traffic flow are stability and steady state flow (6,7). This section is concerned with steady state flow; stability is discussed later.

As mentioned above, various kinds of models can be obtained by determining the values of m and l of the following equation, which is derived from equations (2-2) and (2-3):

$$\ddot{X}_{n+1}(t + T) = \frac{c\dot{X}_{n+1}{}^{m}(t + T)\left[\dot{X}_n(t) - \dot{X}_{n+1}(t)\right]}{\left[X_n(t) - X_{n+1}(t)\right]^{\ell}} \tag{2-4}$$

The steady states of typical models are examined here. Details of each model will be further mentioned in the following section.

Chandler et al. (3, 8)

We can write equation (2-4) in its simplest form, putting $\dot{X} = u$, $\ell = m = o$ (i.e., the model of Chandler et al) and omitting lag T, since we are considering the steady state, as:

$$\frac{du_{n+1}}{dt} = c\,(u_n - u_{n+1}) \tag{2-5}$$

Since $$X_n - X_{n+1} = s_{n+1} = 1/k_{n+1}$$

then $$u_n - u_{n+1} = \frac{ds_{n+1}}{dt} = \frac{-1}{k^2_{n+1}} \frac{dk_{n+1}}{dt} \tag{2-6}$$

where s_{n+1} is the spacing and k_{n+1} the concentration.

The equation can then be written, omitting the subscrips, as:

$$\frac{du}{dt} = \frac{-c_1}{k^2} \frac{dk}{dt} \tag{2-7}$$

Integrating once gives:

$$u = \frac{c_1}{k} + A \tag{2-8}$$

If it is assumed that all the changes of concentration are governed by this equation, and that when the concentration is k_j (jam concentration, vehicles/mile), the flow, and therefore the velocity, is zero, we can evaluate A by using the boundary condition $u = 0$ at $k = k_j$ to give $A = {-c_1}/k_j$ and then:

$$u = c_1(1/k - 1/k_j) = c_1(s - s_j) \tag{2-9}$$

Now, in the steady state $q = uk$

and hence
$$q = c_1(1 - k/k_j) \tag{2-10}$$

where q is the flow rate.

This equation gives the flow-concentration relationship. Note that c_1 is here a flow rate. The quantity k/k_j is the normalized concentration.

Greenberg (1, 7)

Similarly, letting $\dot{X} = u$ and $\ell = 1$, $m = 0$, i.e., $\lambda = c_2/(X_n - X_{n+1})$ and omitting lag T, equation (2-4) becomes

$$\frac{du_{n+1}}{dt} = \frac{c_2(u_n - u_{n+1})}{(X_n - X_{n+1})} \tag{2-11}$$

Omitting the subscript, as

$$\frac{du}{dt} = \frac{-c_2}{k}\frac{dk}{dt} \tag{2-12}$$

which integrates to give

$$\begin{aligned} u &= c_2 \ln \frac{k_j}{k} \\ &= c_2 \ln \frac{s}{s_j} \end{aligned} \tag{2-13}$$

In the steady state $q = uk$, and so the flow-concentration relationship becomes

$$q = c_2 k \ln \frac{k_j}{k} \tag{2-14}$$

where c_2 is here a velocity.

Equations (2-13) and (2-14) are not valid near $k = 0$, so that the constant of integration must be found, in both cases, by using the boundary condition given above. The equations thus apply to dense traffic only.

Edie (5, 7)

Setting $\ell = 2$, $m = 1$, i.e., $\lambda = c_3 \dot{X}_{n+1}/(X_n - X_{n+1})^2$, in (2-4) gives

$$\frac{du}{dt} = -cu \frac{dk}{dt} \tag{2-15}$$

and on integrating,

$$\ln u = -c_4 k + A \tag{2-16}$$

This last equation is not valid near $k = k_j$ (in other words, this case refers to light traffic), so A must be evaluated using the other boundary condition $u = u_f$ at $k = 0$. Hence.

$$\begin{aligned} u &= u_f e^{-ck} \\ &= u_f e^{-c/s} \end{aligned}$$

and then

$$q = u_f k e^{-ck} \tag{2-17}$$

where c is here a vehicle spacing.

Greenshield (9,18)

Finally, setting $\ell = 2$, $m = 0$, i.e., $\lambda = c/(X_n - X_{n+1})^2$, in (2-4) gives

$$\frac{du}{dt} = -c_3 \frac{dk}{dt} \tag{2-18}$$

and on integration,

$$u = -c_3 k + A \qquad (2\text{-}19)$$

But again, $u = uf$ at $k = 0$ so that

$$u - uf = -c_3 k = -c_3 / s$$

and

$$q = k(uf - c_3 k) \qquad (2\text{-}20)$$

Macroscopic Aspect of Car Following Models

The Fundamental Diagram

The relationship between q, the flow of traffic in vehicles per hour, and k, the concentration in vehicles per mile, has been called the fundamental diagram for traffic. It has engaged the interest of traffic engineers for many years.

Since the flow is zero when the concentration is zero, then, if it is assumed that the flow falls to zero at a jam concentration k_j, for values of concentration between these limits the flow must rise to at least one maximum and the shape of the curve must be approximately as shown in Figure 1.

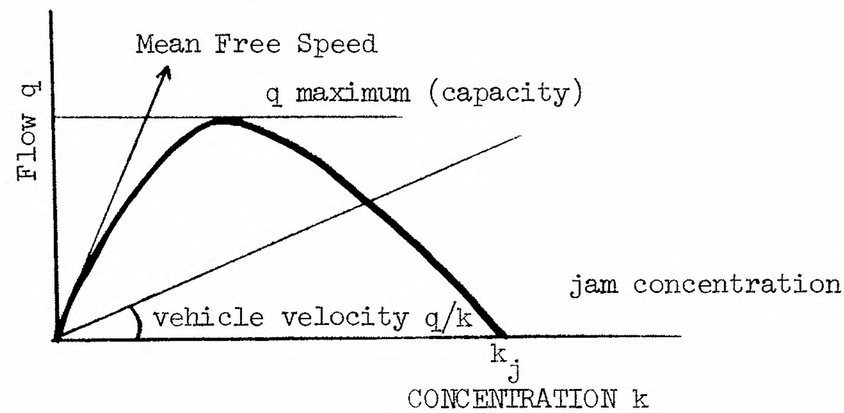


Figure 1. Fundamental Diagram

The Diagram for Each Model (7, 8)

The nature of the steady state of each model which has been mentioned in the previous section can be easily understood by means of diagrams.

Chandler et al. The Chandler et al. Model is $\ell = 0$, $m = 0$ in equation (2-4), i.e., $\ddot{X}_{n+1}(t + T) = c_1\left[\dot{X}_n(t) - \dot{X}_{n+1}(t)\right]$, and its steady state is given as (2-10), $q = c_1(1 - k/k_j)$. A diagram of this model is shown in Figure 2.

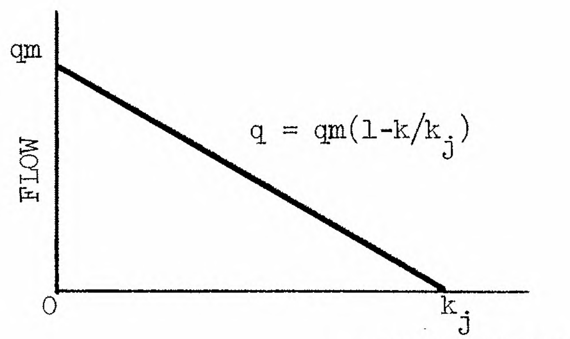


Figure 2. Diagram of Chandler et al. Model.

In this model, coefficient c_1 is a maximum flow rate, qm. As previously mentioned, this model does not fit situations of low concentration; it is not reasonable that the maximum flow rate would be achieved when the concentration is zero.

Greenberg. The Greenberg Model is $\ell = 1$, $m = 0$ in equation (2-4), i.e., $\ddot{X}_{n+1}(t + T) = \dfrac{c_2[\dot{X}_n(t) - \dot{X}_{n+1}(t)]}{[X_n(t) - X_{n+1}(t)]}$, and its steady state is given as (2-14), $q = ck \ln k_j/k$. Then its diagram becomes as shown in Figure 3.

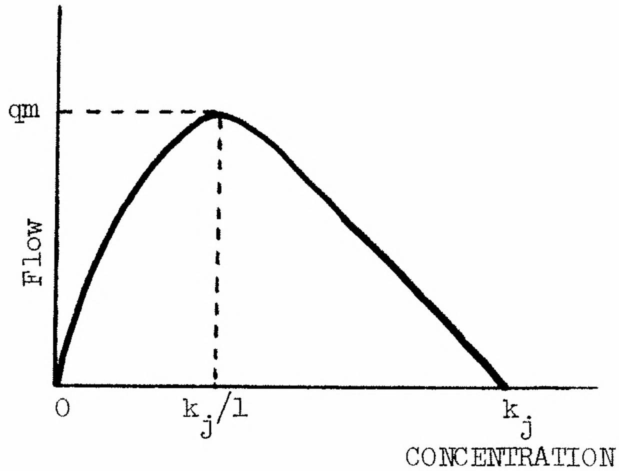


Figure 3. Diagram of Greenberg Model

It is obvious from equation (2-18) that the maximum value of flow, qm, is always achieved when the concentration approaches the value of 36.8 per cent of the jam concentration, i.e., k_j/e, and qm $= V_{op}\ k_j/e$.

Greenshields. The greenshields madel is given as = 2, m = 0 in equation (2-4), i. e.,

$$\ddot{X}_{n+1}(t + T) = \frac{c_3[\dot{X}_n(t) - \dot{X}_{n+1}(t)]}{[X_n(t) - X_{n+1}(t)]^2}$$, and its steady state is

given as equation (2-20), $q = k(uf - c_3k)$.

Its fundamental diagram is shown in Figure 4.

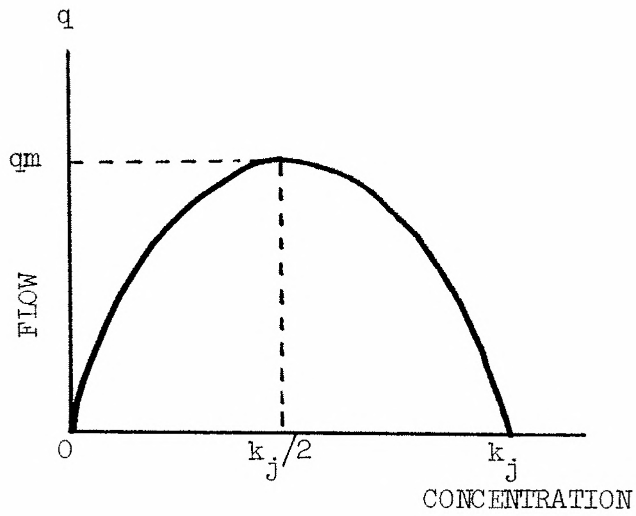


Figure 4. Diagram of Greenshields Model

In equation (2-20), coefficient c_3 must be equal to uf/k_j because q is equal to zero at the concentration k_j (jam concentration); then equation (2-20) can be written as $q = kuf(1 - k/k_j)$. The maximum flow will be achieved at $k_j/2$.

Edie. The Edie Model is given as $\ell = 2$, $m = 1$ in equation (2-4), i.e.,

$$\ddot{X}_{n+1}(t + T) = \frac{c_4 \dot{X}_{n+1}(t)\ [\dot{X}_n(t) - \dot{X}_{n+1}(t)]}{[X_n(t) - X_{n+1}(t)]^2}$$, and its

steady state is given as equation (2-17), $q = k\,u_f \exp(-c_4 k)$.

The fundamental diagram of this model is shown in Figure 5.

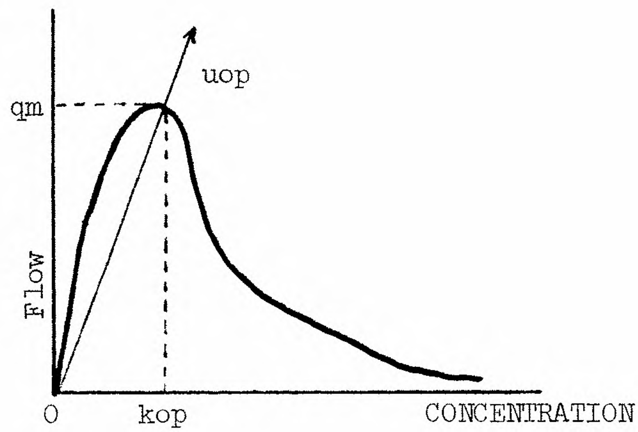


Figure 5. Diagram of Edie Model.

In this model, the maximum flow qm, occurs at the optimum velocity $u_{op} = uf/e$, $k_{op} = 1/c_4$ and $qm = uf/(\lambda e)$.

This model is not valid near $k = k_j$.

Experiments and Validations of Car Following Models

Many experiments have been done for the purpose of validation of models and specification of parameters. There are two principal types of approach. One is the microscopic approach, i.e., research which starts mainly from analysis of the equation of car following theory, such as equation (2-1). The other is the macroscopic approach, which starts with an analysis of the relationship between traffic flow and concentration.

Chandler et al. (3)

R. E. Chandler et al., of the research staff of General Motors Corporation, studied cases of constant sensitivity, i.e., $m = 0$, $\ell = 0$ in equation (2-4), and verified their model by the following experiment.

Experiment. Chandler et al. designed equipment to measure the acceleration and velocity of the following car, the relative velocity of the two cars, and their spacing. A steel wire was connected between the rear of the lead car and a reel fixed to the front bumper of the following car. A friction clutch maintained constant tension in the wire while the cars were in motion. The device gave a continuous record of spacing and relative velocity by means of a potentiometer and tachometer. Longitudinal acceleration was obtained from an accelerometer mounted in the following car, and absolute speed was obtained using a fifth wheel. Extensive experimental runs were carried out on a reserved test track at speeds, varied randomly by several d drivers, from 10 to 80 miles per hour.

Result. Chandler et al. analyzed data obtained by the above experiment to determine the values of the constant $\hat{c}_1$ and $\hat{T}$ which yield the best least squares fit to the equation.

$$\ddot{X}_{n+1}(t + \hat{T}) = \hat{c}_1[\dot{X}_n(t) - \dot{X}_{n+1}(t)] \tag{2-21}$$

Since the reaction time T of each test driver was unknown, by plotting $\hat{T}$ versus the correlation coefficient r, they identified a most likely value for each driver. By this method, they obtained the following results.

The average $\hat{T}$ at maximum r = 1.55 sec

The average $\hat{c}_1$ = 0.368 sec^{-1}

The average of maximum r = 0.80

Then it can be concluded that this model gives a fairly good approximation to the actual car following situation.

Greenberg (1)

H. Greenberg introduced his model from the assumption that traffic behaves like a continuous fluid, and the methods of fluid dynamics may then be used except for the lowest densities of traffic. He started from the equation of motion of a one-dimensional fluid and finally derived the same equation as equation (2-14),

$$q = c_2 k \ln(k_j/k). \tag{2-22}$$

The following equations can be derived from this equation.

$$u = c_2 \ln(k_j/k), \tag{2-23}$$

$$s = s_j \exp(U/c_2), \tag{2-24}$$

where s is headway (ft) between vehicles and

$$s_j = 5{,}280/k_j. \tag{2-25}$$

In order to verify the theory, it is necessary to fit data using the equations presented above. Greenberg applied data which were recorded with a Simplex Productograph Machine in the North Tube of the Lincoln Tunnel. This machine was actuated to record the time when a vehicle passed two observation points along a short length of roadway.

Making a least-squares fit to the data resulted in the headway relation,

$$s = 23.2\exp(u/17.2), \tag{2-26}$$

or for the density, $k = 228\exp(-u/17.2).$ (2-27)

Making a least squares fit to publish data taken at the Meritt Parkway, using fine-minute time profiles, resulted in

$$s = 24.6\exp(u/16.1) \tag{2-28}$$

$$k = 215\exp(-u/16.1). \tag{2-29}$$

In both of the above cases an excellent correlation between theory and empirical results is obtained.

Greenshields

Greenshields made an experimental study of traffic flow by measuring actual flows and velocities of observed vehicles. He plotted the velocity against density for one-lane traffic; he then fitted the points by a straight line, i.e., $u = ak + b$. If a is replaced by -c and b is replaced by uf, we obtain,

$$u = -ck + uf.$$

Then $$q = uk = k(uf - ck),$$ which is exactly the same as equation (2-20).

The same result as Greenshields' model was obtained by Michaels (1963) (9) when considering the visual angle that may be perceived by the following driver, as described below.

From a perceptual standpoint, the transverse location of an object in a driver's path (or the width of the car in front of a driver) may be considered a problem in trigonometry. The transverse distance, W of an object may be derived from the simple trigonometric expression shown in Figure 6:

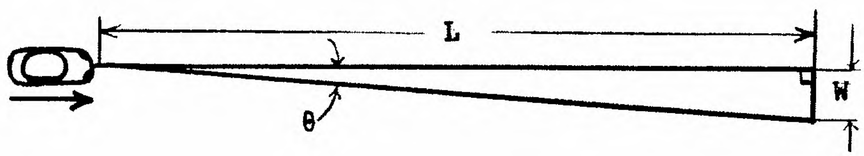


Figure 6. Driver's Perception

Then $$\tan\theta = \frac{W}{L} \cong \theta \quad \text{if } \theta \text{ is small.}$$

$$\frac{d\theta}{dt} \cong - \frac{W}{L^2} \frac{L}{dt} , \tag{2-30}$$

since L is relative spacing and $\frac{dL}{dt} = \dot{X}$ is velocity,

$$\left|\frac{d\theta}{dt}\right| \cong \left|\frac{\dot{X}_n(t) - \dot{X}_{n+1}(t)}{[X_n(t) - X_{n+1}(t)]^2}\right| \tag{2-31}$$

It has been shown (9) that $\left|\frac{d\theta}{dt}\right|$ is below the threshold of relative velocity detection until the quantity on the right-hand side of the above equation exceeds a certain amount. More details about the value are mentioned in the section on Data Collection.

Edie (5)

Leslie C. Edie suggested a variation in various car following theories in an effort to make them more accurate for less than optimum traffic densities. He pointed out the following.

(1) Although the steady state model developed by Herman and Greenberg, i.e., equation (2-14), $u = c \ln (k_j/k)$, has been shown by the latter to fit two sets of experimental data with good agreement, it is obvious that the model becomes less and less realistic as the traffic becomes less and less dense. This loss of realism is exhibited by the lack of an upper limit on the stream velocity, since as $k \rightarrow 0$,

$$u \rightarrow \ln(k_j/0) \rightarrow \infty.$$

One might state that the failure of the model to explain low density is of no great importance. At extremely low density the follow-the-leader theory is not applicable, since there is no interaction between vehicles which would be affected by the leader. However, under such conditions, the stream velocity would approach the mean free velocity of the vehicles and would not increase without limit, as it does in this model.

(2) The refinement to the car following model proposed herein states that the sensitivity of the driver varies with his absolute velocity; the faster he is going, the greater his sensitivity. The factor in the sensitivity situation is also dependent inversely on spacing; if the car ahead is close, the sensitivity to absolute velocity is greater. As the spacing increases, the effect of both velocity and absolute velocity on acceleration would approach zero, a condition that presumably would occur at the free velocity of the car when alone on the road. These further suppositions also seem reasonable in the light of subjective experience.

Then L. C. Edie suggested the following theory:

$$\ddot{X}_{n+1}(t + T) = \frac{c\,\dot{X}_{n+1}(t)\left[\dot{X}_n(t) - \dot{X}_{n+1}(t)\right]}{\left[X_n(t) - X_{n+1}(t)\right]^2} \qquad (2\text{-}32)$$

and its steady state has been obtained as equztion (2-17), which also is written in terms of k, k_{op};

$$k = k_{op}\ln(uf/u), \quad q = uk_{op}\ln(uf/u) \qquad (2\text{-}33)$$

where k_{op} is the optimum concentration, i.e., the value of concentration which maximizes flow. By dividing the previously mentioned data from the Lincoln Tunnel used in Greenberg's paper into two parts the congested flow and the non-congested flow, Edie applied this model to the latter, because as shown in Figure 7, a discontinuity is suggested at a density (concentration between 75 and 100 vehicles per mile.

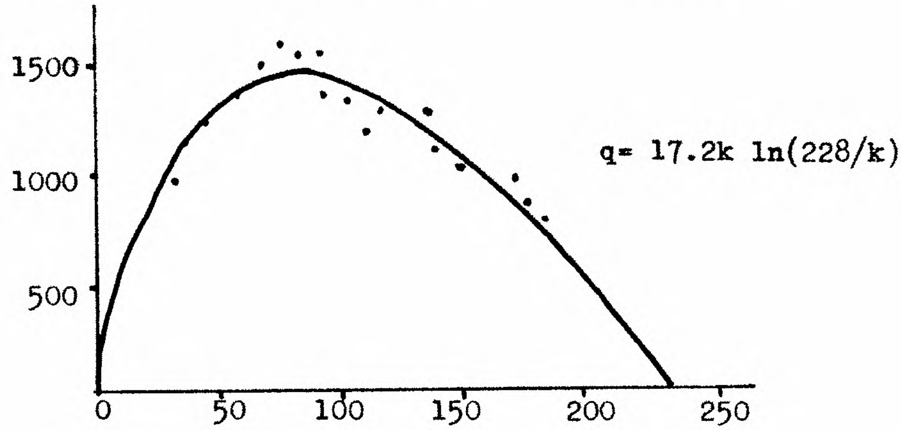


Figure 7. Flow versus concentration k (car/mile) from Greenberg's paper

Applying Greenberg's theory, $q = ckln(k_j/k)$, to the congested flow and Edie's model to the noncongested flow, he obtained:

For congested flow, $q = 14.5kln(250/k)$. (2-34)

i.e., the jam density k_j = 250 vehicles/mile, the optimum velocity c = 14.5 miles/hour, and the maximum flow qm = 1330 vehicles/hour.

For noncongested flow, $q = 90U \ln(46/u)$, (2-35)

i.e., the free velocity u_f = 46 miles/hour, the optimum concentration k_{op} = 90 vehicles/mile, and the maximum flow qm = 1520 vehicles/hour. Edie concluded that comparing Figure 8, which shows the case of division into congested flow and noncongested flow, with Figure 7, the overall fit of the data to the curves appears superior in Figure 8.

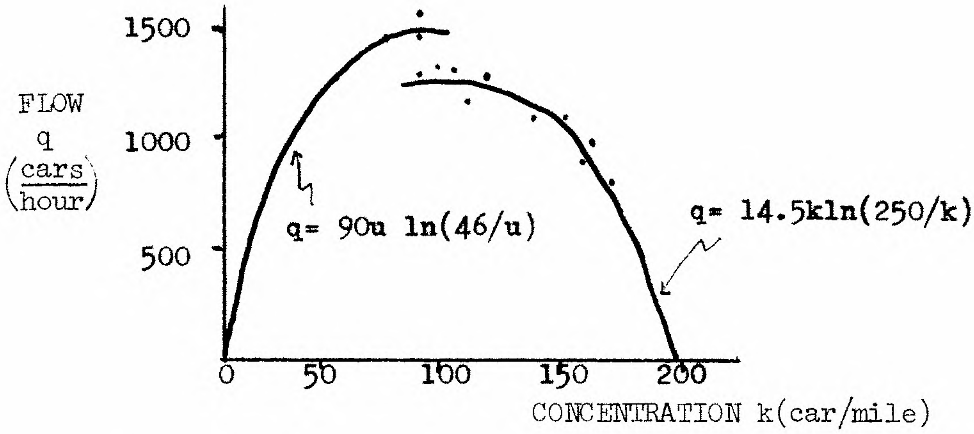


Figure 8. Flow versus Concentration in case of division into congested flow and noncongested flow.

Stability (7, 10)

As mentioned before, stability is a very improtant characteristic of traffic flow; however, unfortunately, the investigation of stability is mathematically difficult for any equations other than the linear system obtained by assuming a constant sensitivity.

Herman et al. (3, 10) investigated the question of stability by introducing a reaction time T into a simple model. Letting $\dot{X} = u$, $\lambda = c$ in equation (2-2) and writing t for (t + T), we get:

$$\frac{du_{n+1}(t)}{dt} = c\left[u_n(t - T) - u_{n+1}(t - T)\right]. \tag{2-36}$$

By using the Laplace Transformation,

$$SU_{n+1}(S) - u_{n+1}(0) = ce^{-ST}\left[U_n(S) - U_{n+1}(S)\right] \tag{2-37}$$

i.e.,

$$\left[S + ce^{-ST}\right]U_{n+1}(S) = u_{n+1}(0) + ce^{-ST}U_n(S). \tag{2-38}$$

The transformed solution, $U_{n+1}(S)$, is then given by

$$U_{n+1}(S) = \frac{U_{n+1}(0) = cU(S)e^{-ST}}{s = cl^{-sT}} \tag{2-39}$$

where $$U_{n+1}(S) = L[u_n(t)] = \int_0^{\infty} u_n(t)e^{-ST}dt. \tag{2-40}$$

There are two kinds of instability in a car following model. One is concerned with the response of one vehicle to its neighbor and has been called local instability. The other one is a phenomenon called asymptotic instability, which is a disturbance that grows as it is transmitted down a line of traffic.

For local instability, the general character of the solution is determined by the zeros of the denominator of the equation (2-39). These occur at the root of

$$S + ce^{-ST} = 0. \tag{2-41}$$

Setting $sT = \alpha + i\beta$ and equating real and imaginary parts, the roots can be found as intersections of curves

$$\alpha + cTe^{-\alpha}\cos\beta = 0, \tag{2-42}$$

$$\beta - cTe^{-\alpha}\sin\beta = 0.$$

For large t, the character of the solution depends on the pole having the largest real part, the contribution to the velocity from the other poles being heavily damped. Even for small t, it can be shown that it is sufficient to consider this pole. The results are then

$$cT \leq \frac{1}{e} = 0.368 \qquad \text{non oscillatory;} \tag{2-43}$$

$$\frac{1}{e} < cT < \frac{\pi}{2} = 1.57 \qquad \text{damped oscillation;} \tag{2-43}$$
$$cT = \frac{\pi}{2} \qquad \text{undamped oscillation;}$$
$$cT > \frac{\pi}{2} \qquad \text{increasing oscillation.}$$

To investigate asymptotic instability, equation (2-36) can be writted as

$$\frac{du_{n+1}(t)}{dt} = c\frac{ds_{n+1}(t-T)}{dt} \tag{2-44}$$

and the right-hand side can be expanded in a Taylor series as far as the term in T^2, thus

$$\frac{du_{n+1}(t)}{dt} \cong c\left[\frac{d}{dt}\, s_{n+1}(t) - T\frac{d^2}{dt^2}\, s_{n+1}(t) + \frac{T^2}{2}\,\frac{d^3}{dt^3}\, s_{n+1}(t)\right]. \tag{2-45}$$

Now since
$$s_{n+1} = (X_n - X_{n+1})$$

then
$$\frac{ds_{n+1}}{dt} = (u_n - u_{n+1})$$

$$\frac{d^2 s_{n+1}}{dt^2} = \frac{d}{dt}(u_n - u_{n+1}) \;\cdots\cdot \tag{2-46}$$

Hence, collecting terms in u_{n+1} on the left-hand side,

$$\frac{cT^2}{2}\,\frac{d^2{}_{n+1}}{dt^2} + (1 - cT)\frac{du_{n+1}}{dt} + cu_{n+1} = \frac{cT^2 d^2 u_n}{2dt^2} - cT\frac{du_n}{dt} + cu_n \tag{2-47}$$

If it is assumed that $u_n = U_n e^{i\omega t}$, then it is found that differentiating and substituting in

$$U_{n+1}\left[-cT^2\omega^2/2 + (1 - cT)i\omega + c\right] = U_n\left[-cT^2\omega^2/2 - cTi\,\omega + c\right] \tag{2-48}$$

and this implies that u_{n+1}/u_n grows as n increases if

$$\frac{1 - cT}{cT} < 1 \quad \text{or} \quad cT > \frac{1}{2}. \tag{2-44}$$

It is difficult to solve the stability of the Greenberg model because of its non-linearity. However, the stability can be inferred as follows:

In the case of the Chandler et al. model, the system is asymptotically stable if

$$c_1 T < \frac{1}{2},$$

where both c and T are constant, c is the sensitivity coefficient and T is reaction time or time lag.

On the other hand in the case of the Greenberg model, the system is asymptotically stable if

$$\frac{c_2 T}{D} < \frac{1}{2}$$

where D is the distance between the front car and the driver's own car and it is a variable.

The differences of the two models relative to stability can be seen in Figure 9.

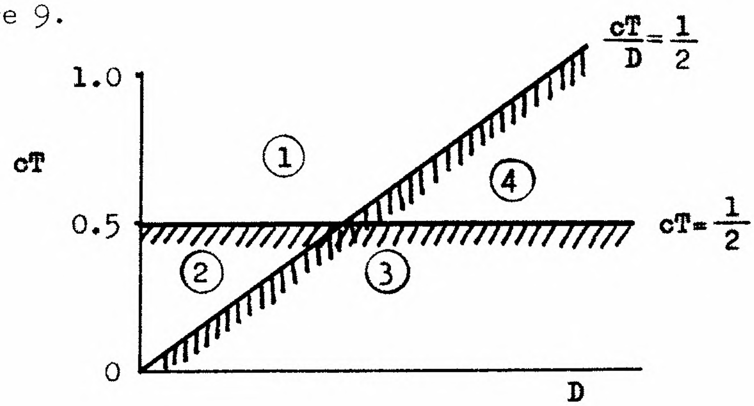


Figure 9. Stability Region

Regions 2 and 3 are stable in the Chandler et al. model regardless of the distance between the two cars; however, in the Greenberg model, regions 3 and 4 become the stable regions.

Thus, the stable regions which vary are regions 2 and 4. The Greenberg model is preferable from the point of view of stable regions because if the distance between two cars is increasing, the interaction between the cars must be getting smaller and stability must increase, as in region 4; conversely, as the distance decreases, the stability must decrease, as in region 2.

Summary of Mathematical Models

The mathematical models that have been described can be represented by the general response equation (2-2)

$$\ddot{X}_{n+1}(t + T) = \lambda(\dot{X}_n(t) - \dot{X}_{n+1}(t))$$

where λ is the sensitivity (2-3)

$$\lambda = \frac{cX^m_{n+1}(t + T)}{\left(X_n(t) - X_{n+1}(t)\right)^{\ell}}$$

and $X_n(t)$ is the location of nth car at time t,

$\dot{X}_n(t)$ is velocity of nth car at time t,

$\ddot{X}_{n+1}(t)$ is acceleration of n+1 th car at time t,

T is reaction time,

λ is sensitivity, c, ℓ, and m are constants.

The models can be classified by the values for the constants ℓ and m. The resulting specific response equations and steady-state equations are then as given in Table 1. Stability conditions have been determined mathematically only for the Chandler et al. model, since this is the only linear model. These are also included in Table 1.

The fundamental diagram represents a macroscopic description of the relation between the vehicle flow rate and the concentration. Figures 2-5 present the diagrams for the four mathematical models. These are presented again here for convenience.

It can be seen that the various models exhibit significant differences in the characterization of traffic flow, both from the microscopic (Table 1) and the macroscopic (Figures 2-5) viewpoints. In Chapter III the merits of each model will be examined in more detail, so that one may be selected for further development. The criterion for selection will be based on a combination of factors: 1) microscopic behavior of the human-mechanical system, 2) observed macroscopic behavior of car platoons.

Table 1. Summary of Mathematical Models

Name	ℓ	m	response equation	
Chandler et al.	0	0	$\ddot{X}_{n+1} = c_1(\dot{X}_n - \dot{X}_{n+1})_{t-T}$	$q = c_1(1-k/k_j)$
Greenberg	1	0	$\ddot{X}_{n+1} = \frac{c_2(\dot{X}_n - \dot{X}_{n+1})}{(X_n - X_{n+1})_{t-T}}$	$q = c_2 k \ln k_j/k$
Greenshields	2	0	$\ddot{X}_{n+1} = \frac{c_3(\dot{X}_n - \dot{X}_{n+1})}{(X_n - X_{n+1})^2_{t-T}}$	$q = k(u_f - c_3 k)$
Edie	2	1	$\ddot{X}_{n+1} = \frac{c_4 \dot{X}_{n+1}(\dot{X}_n - \dot{X}_{n+1})}{(X_n - X_{n+1})^2_{t-T}}$	$q = u_f k \exp(-c_4 k)$

Stability of Chandler et al. Model

c_1T	local	asymptoic
$c_1T \le 1/e$	non-oscillarory	non-growing
$1/e < c_1T < 1/2$	damp-oscillatory	non-growing
$1/2 \le c_1T < \pi/2$	damp-oscillatory	growing
$\pi/2 < c_1T$	increasing-oscil	growing

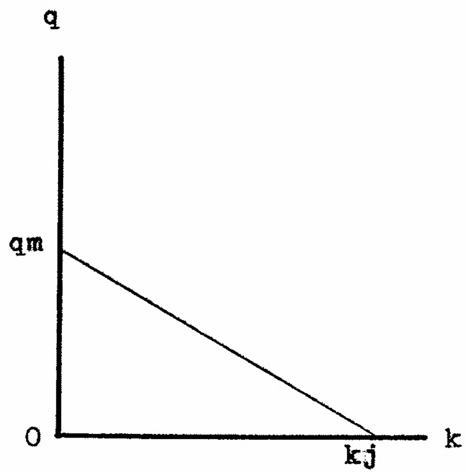


Figure 2. Chandler et al.

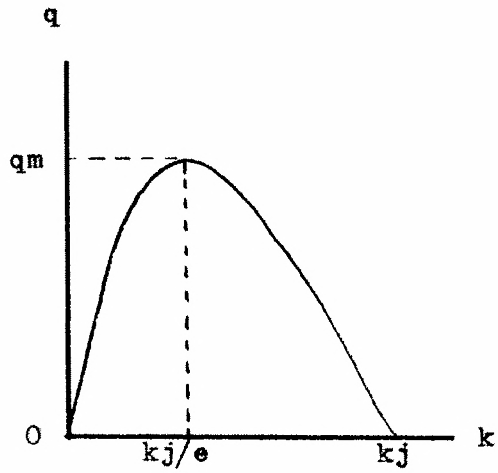


Figure 3. Greenberg

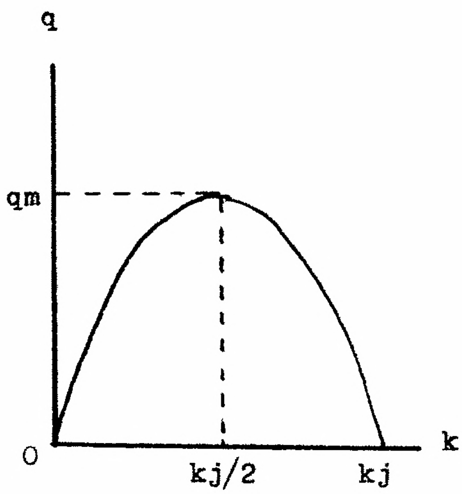


Figure 4. Greenshields

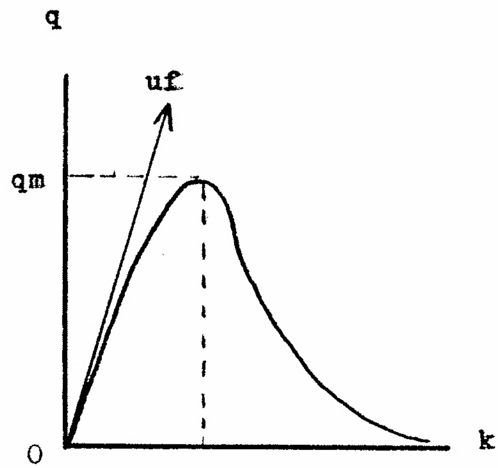


Figure 5. Edie

q is flow rate
k is concentration

qm is maximum flow rate
kj is jam concentration
uf is free velocity

Fundamental Diagram

CHAPTER III

THE SIMULATION MODEL

The Importance of Simulation

Mathematical car following models have greatly contributed to an understanding of traffic flow and the interactions among cars in a platoon. However, these contributions can be said to be rather conceptual because these mathematical models are very useful for understanding the general concept of traffic and car following, but they lack adaptability and reality in the following aspects:

(1) Difficulty in the setting of limits or addition of important factors, for example:

(a) Acceleration and deceleration performance of a car has limits, and in actuality linear relationships cannot always exist.

(b) The effect of brake lights is significant on the driver's reaction time.

(c) Other important factors which affect the driver's behavior (i.e., response) other than sensitivity and stimulation, for example, each driver's desired distance from the car which travels in front of him.

(2) The analytic intractability of extended mathematical models which include the above-mentioned factors.

Simulation models, on the other hand, can be used effectively to provide structural understanding and reality to the parameters, while at the same time including the various model components in (1) above. Furthermore, simulation is a very effective method of analyzing traffic problems. In most cases, it is difficult to carry out experimentation since sometimes too many complicated factors are involved and sometimes it is dangerous or very expensive.

Selection of Simulation Language

It is necessary to select one of the continuous flow simulation languages for car following models because the characteristic of a car following model is described by a differential equation, such as equation (2-4). DYNAMO (2), which represents an Euler scheme of numeric integration (derivatives held constant for small intervals), is an effective language for the purpose of this thesis.

DYNAMO is the simulation language which has been developed by J. W. Forrester, W. R. Fey, et al. for the purpose of modeling systems according to the methodology of Industrial Dynamics (11) - also called Feedback Dynamics or Systems Dynamics.

The description of DYNAMO syntax allows the following types of equations:

i) Level equation, also called accumulation, which defines the volume of the level at a specific time point k. Typically, it has the following format:

LEVEL.K=LEVEL.J+(DT)(INPUT.JK-OUTPUT.JK) (3-1)

where J, K are suffixes which express successive points in time, JK is a period of time of length DT, and DT is the simulation time interval. Thus, level equations can be used to represent integral equations, or, from another viewpoint, differential equations.

ii) Rate equation, which defines the rate of inflow or outflow for a specific accumulation. An example of its format is:

$$\text{INPUT.KL}=(0.5)(\text{LEVEL.K}) \qquad (3\text{-}2)$$

where the suffix KL represents a period of time, again of length DT.

iii) Auxiliary equation, which helps to define more detailed or complicated relationships between equations.

Model Building

To develop a model which is flexible enough to be usable, it is necessary to verify that the model is correct and that it contains all important factors.

Determination of Type of Reaction Delay

The reaction time is one of the most important factors in a car following model; it is written as T in the equations of Chapter II. As mentioned in the Traffic Engineering Handbook (12) the reaction time is understood as the time interval between the stimulus and the response. Its components are perception, interpretation, decision making, and response time.

Industrial Dynamics practitioners approximate time lags with standard DYNAMO delay functions. A first order time delay or a third-order delay is usually applied, while a pipeline delay is seldom used.

The determination of which type of delay to be used is a significant problem in the car following model, because the method of representing a differential equation by DYNAMO is an approximation and the delay, i.e., the reaction time, has a great influence on system

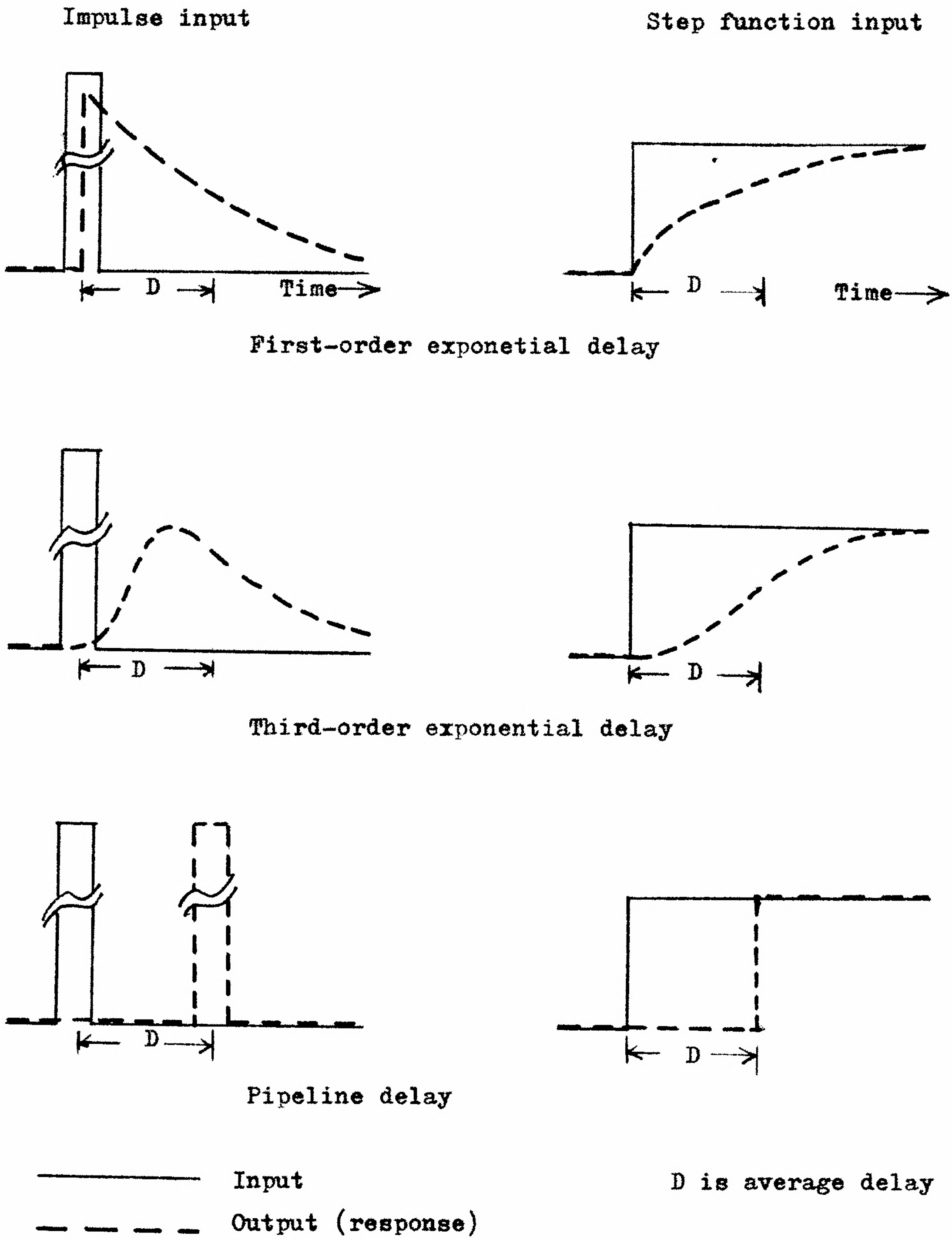


Figure 10. Types of Delay and Response from (11).

stability, as shown later.

The DYNAMO delays are characterized by the average time lag and the dispersion. As an example, Figure 10 displays the different dispersions produced when the input flow is (a) an impulse and (b) a step function.

Examination of Type of Delay. The first criterion in selecting the type of delay is how precisely that delay describes an actual reaction time in car following. The second criterion is ease of dealing with the type of delay.

Let us examine the characteristics of the reaction time itself in order to check it from the standpoint of the first cirterion. The characteristics of the reaction time might be inferred by assuming the situation in which the driver responds when the car in front of him brakes hard for a very short time and then resumes its previous speed (i.e., impulse stimulus) or brakes hard and decreases its speed by some degree (i.e., step function stimulus). The characteristics of the reaction time are:

(a) There must be a blank interval from the stimulus to the beginning of the response, because the reaction time involves certain minimum times for perception, interpretation, decision making, and response.

(b) However, the shape of the response curve may be different from that of the stimulus input, because of the existence of a filter in the driver's reaction process. Consequently, under the first criterion, the third-order delay or pipeline delay are possible choices for reaction time in the car following model.

Next, let us check the second criterion. It is not easy for DYNAMO, (perhaps not for any simulation language) to retain information about an event and retrieve it arbitrarily. Then from the standpoint of the second criterion, the pipeline delay is not as desirable as the first-order delay or third-order delay. The first-order delay is the best type according to the second criterion.

Previous Research and Experiments about Reaction Time. In the traffic engineering literature, there is little detailed research on reaction time because there are too many psychological factors to be handled.

Chandler et al. assumed pipeline type delay as reaction time in their analysis of their experiment, previously described in Chapter II. They defined reaction time as the time interval which maximizes the correlation coefficient between the stimulus (i.e., difference in speed between the driver's car and the car in front of him and the response (i.e., driver's response). In this manner they obtained a high correlation coefficient.

Stability and Type of Delay. In order to examine the influence of type of delay on stability, the Chandler et al. model has been chosen. This is the simplest car following model and all stability conditions can be solved mathematically, as mentioned in Chapter II.

The stability of the Chandler et al. model, i.e.,

$$\ddot{X}_{n+1}(t + T) = c_1[\dot{X}_n(t) - \dot{X}_{n+1}(t)], \tag{3-3}$$

is summarized in Table 1, part of which is given here:

The Value of cT	Local Stability	Asymtotic Stability
$c_1T \leq 1/e$	non-oscillatory	non-growing
$1/e < c_1T \leq 1/2$	damp-oscillation	non-growing
$1/2 < c_1T < \pi/2$	damp-oscillation	growing
$\pi/2 < c_1T$	increasing-oscillation	growing

R. Herman et al., have demonstrated the results of several calculations concerned with the stability of this model in their paper (10). These results are shown as Figures 11 ~ 13.

<u>Simulation of the Chandler et al. Model to Determine the Type of Delay</u>. The stability of the Chandler et al. model depends on the value of c_1T, where T is reaction time and c_1 is a constant which means sensitivity. The critical values of c_1T are 1/e, 1/2 and $\pi/2$, particularly $\pi/2$, because when c_1T is equal to $\pi/2$, undamped-oscillation is obtained as in Figure 12. ($c_1 = 1.57$).

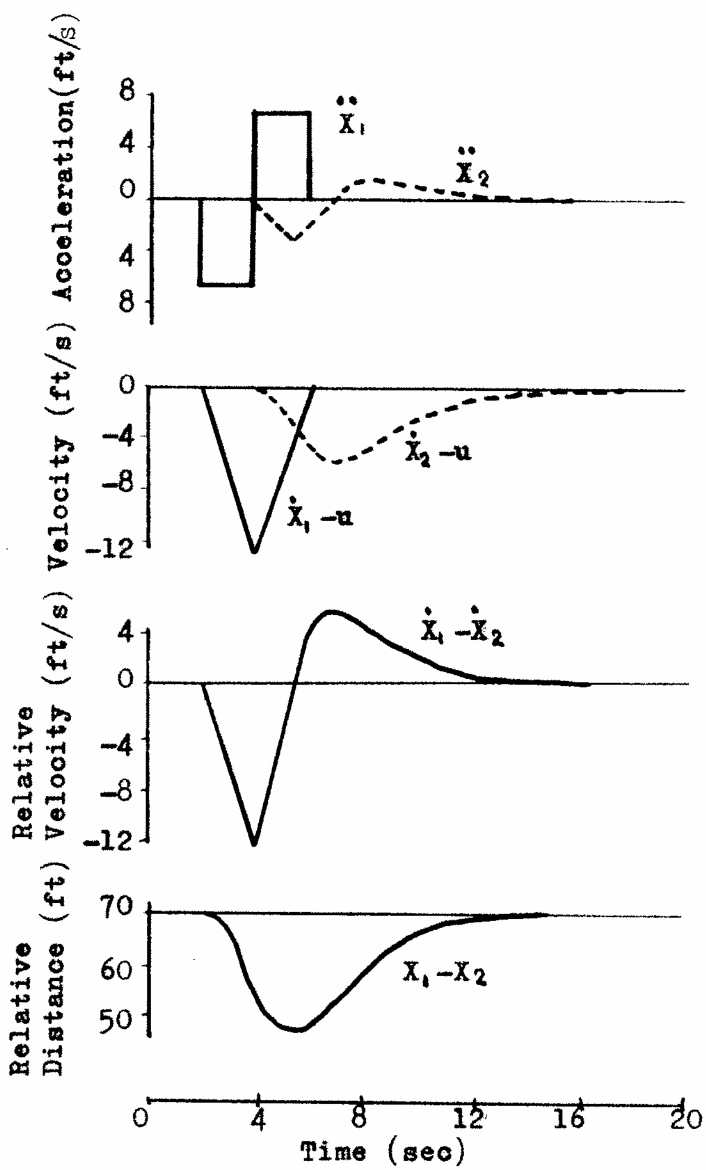
The Chandler et al. model was programmed in DYNAMO and run under the conditions of the following combinations of c_1T, and type of delay in order to examine the difference caused by type of delay.

The runs were executed under almost the same conditions as in Herman's paper (10) for case in comparing the DYNAMO model result and the mathematical result. The initial conditions are as follows:

(a) All four cars travel at the same speed: 90 ft/sec ($\cong$ 60 miles/hour)

(b) Spacing between cars is 70 ft.

(c) The maneuver of the first car is: after one second the



where $c = 1/e = 0.368$
$T = 1.5$ (sec)

Figure 11. Behavior in Chandler et al. Model from (10)

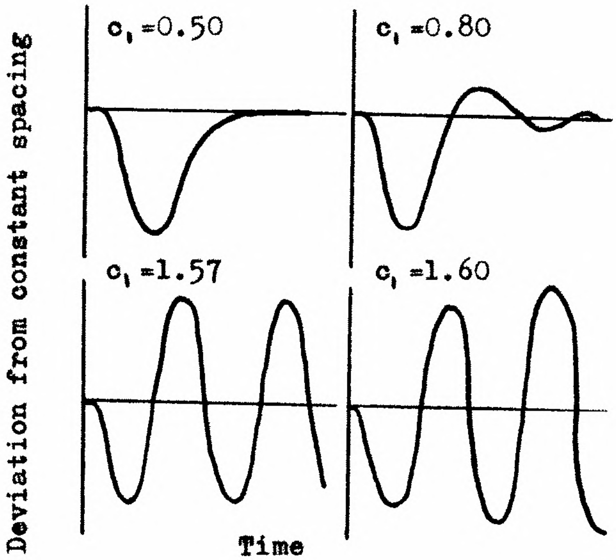


Figure 12. Local Stability of Chandler et al. model from (10).

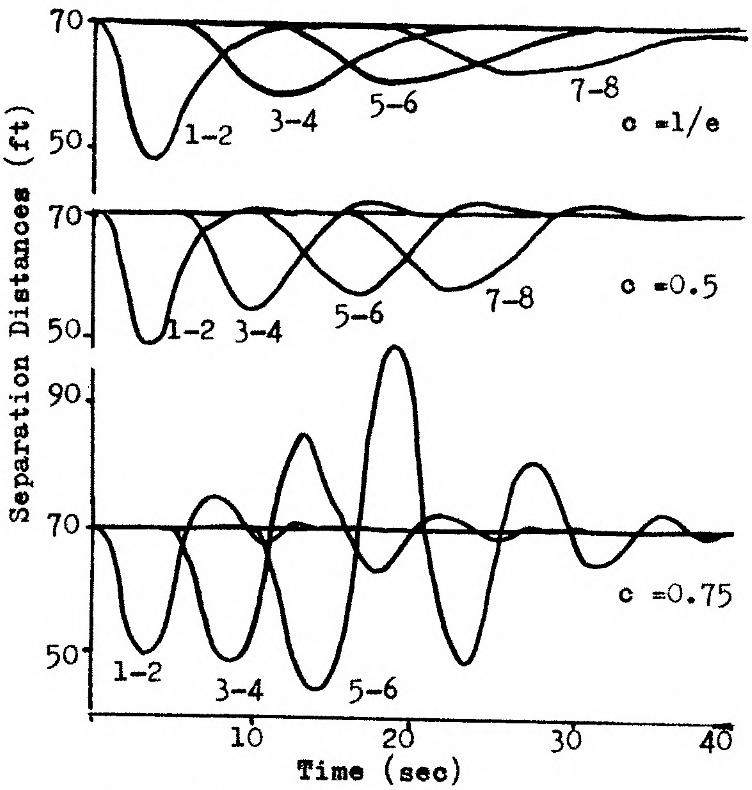


Figure 13. Asymptotic Stability of Chandler et al. model from (10).

Table 2. Combinations of c_1 and T

value of c_1T	T = 1.0	T = 1.5
1/e	$c_1 = 0.368$	$c_1 = 0.245$
1/2	$c_1 = 0.500$	$c_1 = 0.333$
$\pi/2$	$c_1 = 1.571$	$c_1 = 1.047$
2	$c_1 = 2.00$	$c_1 = 1.333$

Types of delay:

first-order delay

third-order delay

pipeline delay

Table 3. Results of Simulations-Comparisons of Stability by Type of Delay

Value of C_1T	Type of Delay	T = 1.0 Local	T = 1.0 Asymptotic	T = 1.5 Local	T = 1.5 Asymptotic
$C_1T = \frac{1}{e}$	mathematical*	non-oscillatory	non-growing	non-oscillatory	non-growing
	1st order	non-oscillatory	non-growing	non-oscillatory	non-growing
	3rd order	non-oscillatory	non-growing	non-oscillatory	non-growing
	pipeline	non-oscillatory	non-growing	non-oscillatory	non-growing
$C_1T = \frac{1}{2}$	mathematical	damp-oscillation	non-growing	damp-oscillation	non-growing
	1st order	damp-oscillation	non-growing	damp-oscillation	non-growing
	3rd order	damp-oscillation	non-growing	damp-oscillation	non-growing
	pipeline	damp-oscillation	non-growing	damp-oscillation	non-growing
$C_1T = \frac{\pi}{2}$	mathematical	undamp-oscil.	growing	undamp-oscil.	growing
	1st order	damp-oscillation	growing	damp-oscillation	growing
	3rd order	damp-oscillation	growing	damp-oscillation	growing
	pipeline	undamp-oscil.**	growing	undamp-oscil.	growing
$C_1T = 2 > \frac{\pi}{2}$	mathematical	increasing-oscil.	growing	increasing-oscil.	growing
	1st order	damp-oscillation	growing	damp-oscillation	growing
	3rd order	undamp-oscil.**	growing	undamp-oscil.**	growing
	pipeline	increasing-oscil.	growing	increasing-oscil.	growing

Note: The lead car's maneuver is the same as in Figure 11.

* Chandler et al. mathematical analysis

**Not exactly undamped-oscillation

first car decelerates at a rate of 6 ft/sec^2 for two seconds, then accelerates at 6 ft/sec^2, returning to its pervious speed of 90 ft/sec^2.

The results of the simulations are shown in Table 3 and in the figures in Appendix C.

As shown in Table 3, the asymptotic stability of the models which are written in DYNAMO coincides with that of the mathematical model; however, the same result as the mathematical solution cannot be obtained from any type of DYNAMO model except in the case of the pipeline delay and a reaction time of 1.5 seconds. This conclusion is drawn from the boundary point of local stability, i.e., the appearance of undamped oscillation for $c_1T = \pi/2$.

Even in this case, the simulation results do not duplicate exactly the mathematical solution. The reason for the difference is that DYNAMO simulates differential equations by approximation, using a specified time interval which is called DT.

The detailed shapes of the output are shown in the figures of Appendix C. In the cases of first-order delay and third-order delay, the starting point of the second car's acceleration are influenced by the value of c_1, sensitivity, and they sometimes begin earlier than the specified delay, while the pipeline delay shows (i.e., RUN DSR-21 in Appendix C) almost exactly the same shape as Figure 11, and the top of Figure 13.

In summary, we can conclude that pipeline delay has the ability to describe almost the same car following behavior as the mathematical model; the third-order delay is not as good as the pipeline delay, but it may be applicable in certain cases; the first-order delay, on the other hand, is of little practical use here.

Determination of the Basic Model

The matter to be determined next is what model is the best basic model to be developed. Each author has insisted on the correctness of his model, as shown in (1), (2), (5), (10) and Chapter II. The model developed by W. Helly in his dissertation was based on the Chandler et al. model; P. Fox and F. G. Lehman (13) developed the Edie model for their simulation studies.

After a detailed comparison of basic models, the following conclusion can be drawn: in congested traffic the Chandler et al. model and the Greenberg model fit well, and in non congested traffic, the Edie and Greenshields models are better.

This fact can be inferred from the process of setting up the equation in Chapter II. Furthermore, the following observations support this conclusion:

(i) J. E. Tolle (8) has compared car following models with empirical data obtained through aerial photogrammetric techniques, and has considered the possibility of using composite models to describe the real world situation. His paper contains observations as shown in Table 4.

(ii) Edie (5) identified the congested flow as $100 < k$ and the non congested flow as $k < 75$ vehicles/mile. (For details, refer to Chapter II.)

Table 4. Regions of Concentration and Average Stream Parameters

Region	No. of observed vehicles	Concentration range (veh/mile)	Average Concentration	Average speed (m.p.h.)	Average flow (veh/hr)	Models with good fit
1	89	$k \leq 40$	27.5	60.2	1650	Edie and Greenshields
2	68	$40 < k \leq 70$	62.4	29.2	1850	none
3	117	$70 < k \leq 140$	98.8	14.8	1460	Chandler et al.
4	41	$140 < k$	180.9	1.4	250	Greenberg

From the above-mentioned conclusion it can be said that the Chandler et al. or Greenberg model is preferable to the Edie and Greenshields models because the simulation model which is to be built is supposed to deal with a car following situation in which rather strong interactions exist. Another reason for prefering the first two models is that they originally were drawn from car following differential equations, while the latter have been developed from the steady state conditions, because for the purpose of this model, the transient situation is more important than the steady state.

Then which is better, the Chandler et al. model or the Greenberg model?

The answer to this question can be found in (4), (6), and (14). D. C. Gazis et al., members of the General Motors Corporation reseach staff, reexamined the car following experiment report which is mentioned in Chapter II in the light of the reciprocal spacing model, i.e., the Greenberg model. Figure 14, taken from their paper, gives a plot of c_1 against the reciprocal of the average spacing s. The straight line is a least-squares fit (excluding the encircled point) for a straight line passing through the origin. It can be seen from this figure that there is a trend for sensitivity to decrease with increasing distance, thus suggesting a nonlinear model such as the Greenberg model.

R. Herman and R. B. Potts (14) have determined the correlation coefficient r versus time lag T when two particular models are applied to data from their observations in the Lincoln Tunnel, as shown in Figure 15.

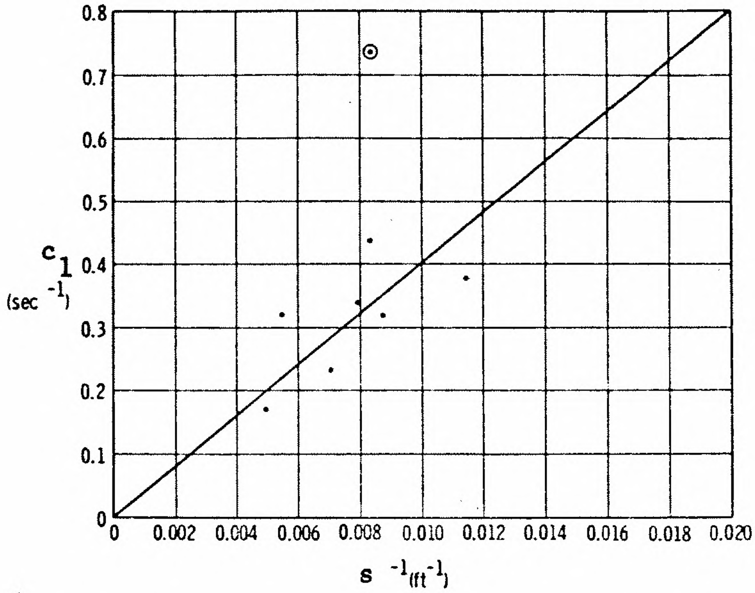


Figure 14. Sensitivity, c_1, veresus Reciprocal of Car Spacing 1/s.

The least squares straight line is represented by $c_1 = 40.2/s$. The encircled point was not included in the fit.

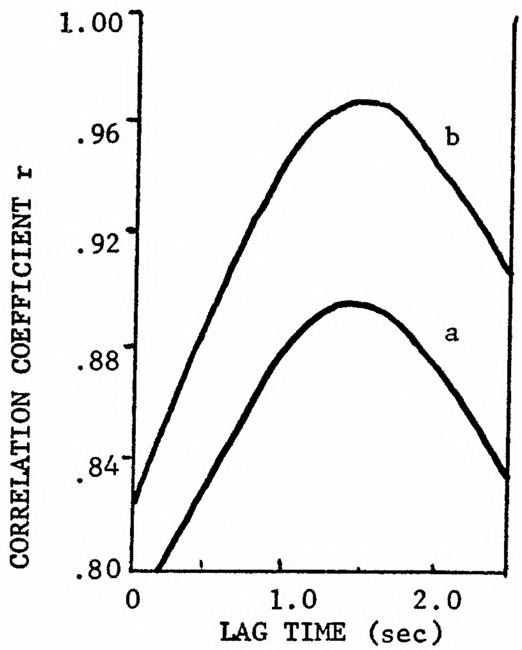


Figure 15. Correlation coefficient, r, veresus Time Lag T.

Curve (a) corresponds to Chandler et al. model, (b) to the reciprocal spacing model, i.e., Greenberg model, from (14).

During the observations, the car spacing varied between 162 feet and 50 feet. The result indicates a time lag T of 1.6 seconds, corresponding to the maximum correlation, and the correlation coefficient is significantly greater for the reciprocal spacing function than for constant c_1. The value 0.97 for the maximum value of r is remarkably close to unity, giving an indication of the accuracy of the car following law. Each period of observation, when analyzed this way, gives an estimate of the coefficient of the reciprocal spacing model, i.e.,

$$\ddot{X}_{n+1}(t + T) = \frac{c_2\left[\dot{X}_n(t) - \dot{X}_{n+1}(t)\right]}{X_n(t) - X_{n+1}(t)} ,$$

The values of c_2 for various observations are shown in Table 5.

The above analysis gives a direct test of the dependence of c_2 on spacing, s, for a given driver. From the results of almost every period of observation, the reciprocal spacing model gives the higher correlation coefficient r, in spite of the variety of observed conditions.

From the foregoing examination of models, we can conclude that the Greenberg Model is the best basic model for development.

Structure of the Simulation Model

The basic structure of the simulation model is shown in Figure 16 and 17. As shown in Figure 16, there are two negative feedback loops for each following vehicle; one is the velocity control loop and the other is the headway control loop. Then, this system has an oscillatory nature. Another characteristic of this system is that closed loops exist only within one vehicle and influences are transmitted from

Table 5. Reciprocal Spacing Coefficient, form (14)

Locality	number of runs	c_2 ft/sec	T sec
GM test track (1)	8	40.2	1.5
GM test track (2)	10	121.5*	0.6*
Lincoln Tunnel	16	29.8	1.2
Holland Tunnel	10	26.5	1.4
Queens Mid-town Tunnel	4	32.0	0.8
number of runs weighted mean		50,1	1.13
		31.5**	1.27**

NOTE: *: GM test track observations (2) involved a sharp maneuver by lead car.

**: mean excludes GM test track (2) data.

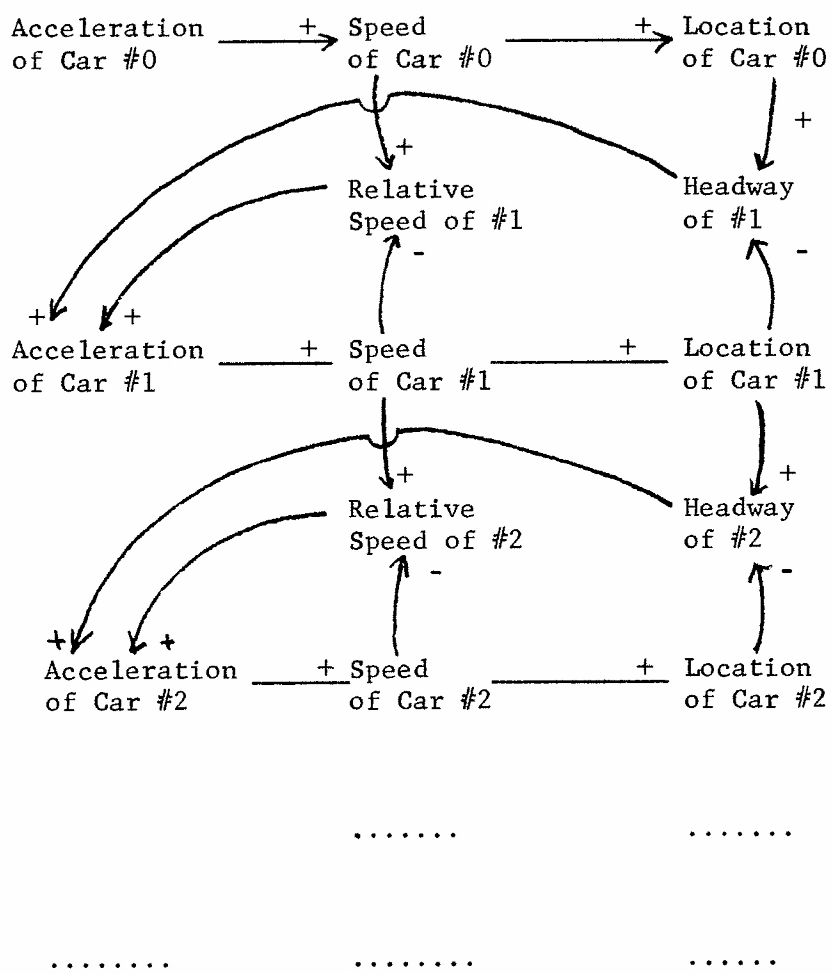


Figure 16. Causality Diagram

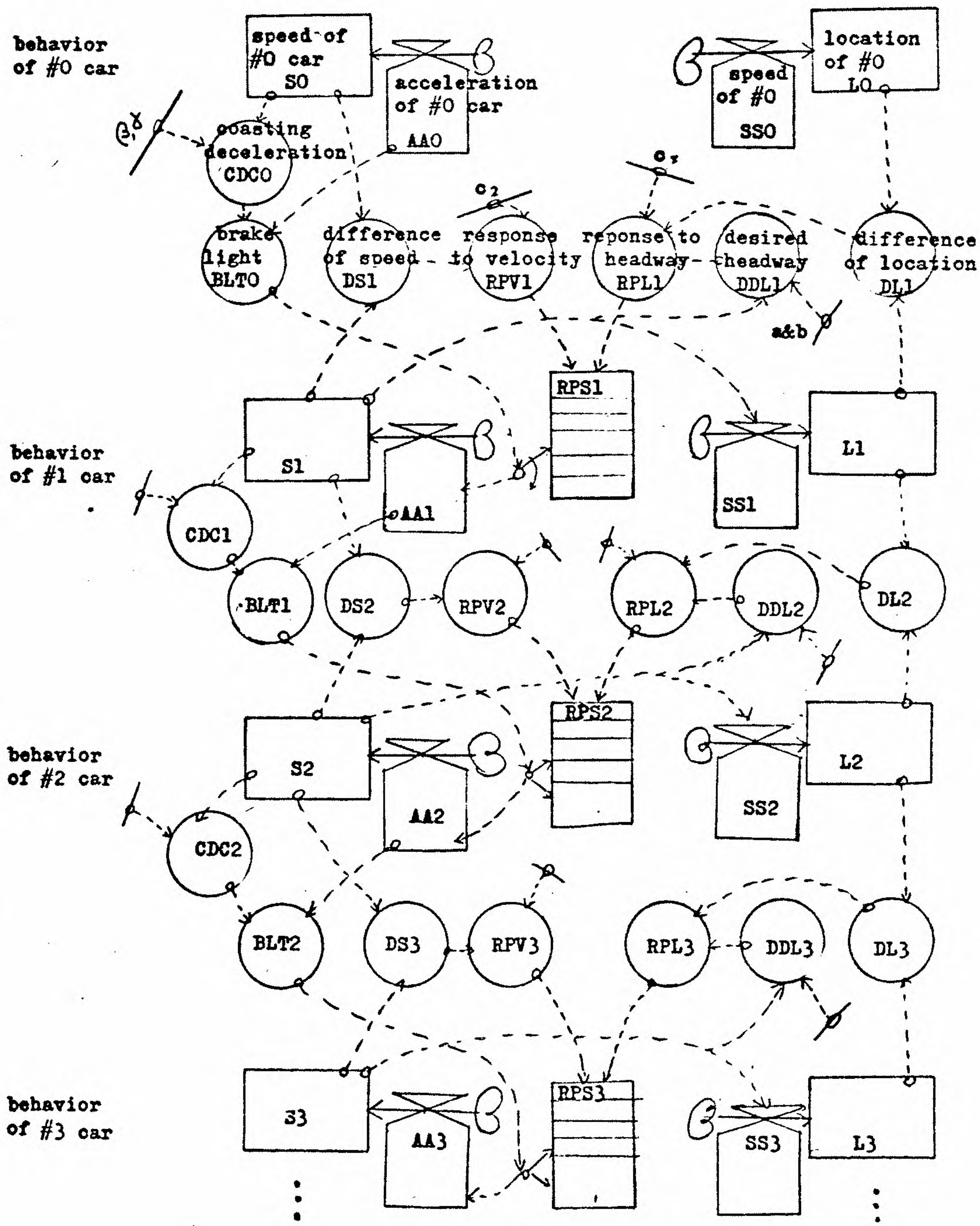


Figure 17. Flow Diagram

one car to another only in a backward direction.

Additional Important Factors, Data Collection

The most important factors are sensitivity and reaction time; these have been examined mainly with the use of mathematical models. However, some additional important factors must be considered when simulation of a car following situation is attempted.

They are: car performancd limits - in the previously examined models, of the car, i.e., acceleration and deceleration, is assumed to be infinite. It is limited in actuality; furthermore, as P. A. Lewis (15) has pointed out, limiting the capacity increases the stability of a platoon.

The effect of brake lights - brake lights obviously improve the reaction time of the driver who sees that they are on. These examples can be seen in the Traffic Engineering Handbook (12) and on page 137 of (14).

Desired distance - in mathematical models, when we specify the speed, headway, and sensitivity coefficients as initial conditions (which can sometimes be equivalent to specifying the jam concentration and/or the free speed) then a collision could occur when the first car stops even though initial conditions are specified to satisfy a stable condition. However, in actuality the driver may control his car by another factor, i.e., the desired distance. The California Vehicle Code states that the nth car should attempt to maintain a desired headway,

$$D = \alpha + \beta \dot{X}_{n-1} \tag{3-4}$$

where α and β are constants.

The Simulation Model Equation. At this point, the problem is how these factors should be combined into the basic model, the Greenberg model. W. Helly (16) has suggested the following equation:

$$\ddot{X}_{n+1}(t + T) =$$

$$c_1\left[\dot{X}_n(t)-\dot{X}_{n+1}(t)\right] + c_2\left[X_n(t)-X_{n+1}(t)-D\right] + c_3B_n(t) + c_4B_{n-1}(t) \quad (3\text{-}5)$$

where X_n = location of the nth car

$\dot{X}_n$ = velocity of the nth car

$\ddot{X}_n$ = acceleration of the nth car

t = time

T = driver reaction time

c_1 = velocity control parameter, $c_1 > 0$

c_2 = headway control parameter, $c_2 > 0$

c_3 = brake factor related to car n

c_4 = brake factor related to car n-1

D = the desired headway for car n+1

$$B_n = \begin{cases} 0 \text{ if car n is not braking} \\ 1 \text{ if car n is braking} \end{cases}$$

$$B_{n-1} = \begin{cases} 0 \text{ if car n-1 is not braking} \\ 1 \text{ if car n-1 is braking} \end{cases}$$

In his equation, the assumption of a linear combination of the velocity difference and the difference between the driver's actual headway and his desired headway seems to be appropriate because they might work independently; it does not make much sense for the terms concerned with the brake factor to be in linear relation.

Consequently, the model which is suggested in this thesis is as follows:

$$\ddot{X}_{n+1}(t + T) = \frac{c_2\left[\dot{X}_n(t)-\dot{X}_{n+1}(t)\right]}{\left[X_n(t) - X_{n+1}(t)\right]} + c_5\left[X_n(t)-X_{n+1}(t)-D\right] \tag{3-6}$$

where $T = T_1$ if nth car's brake lights are not on
T_2 if nth car's brake lights are on

$$T_2 \leq T_1$$

$D = \alpha + \beta\dot{X}_{n+1}(t)$; desired distance.

The difference between this model and the Helly model is that the basic model which has been selected in this research is the Greenberg model, while Helly applied the Chandler et al. model and formulated the effect of brake lights. The reasons why the Greenberg model seems to be more appropriate have been mentioned before.

The following sections describe the data collection necessary to implement the simulation of equation (3-6).

Reaction Time and Sensitivity Coefficient. If reaction time is defined as the time interval from the stimulus to response, then it is considered that reaction time consists of perception, interpretation, decision making and response. However, it is extremely difficult to subdivide reaction time into these factors. Brake reaction time is shown in Table 3, 9, page 82 of the Traffic Engineering Handbook (12), which is reproduced here as Table 6. This table indicates that the brake reaction time is about 1.65 seconds without a stop light and about 0.82 seconds with a stop light under normal road conditions.

Another reaction time observation, which includes the effect of

Table 6. Brake Reaction Time Measured Under Varying Conditions from (12)

Car Movement	Stimulus	Starting Foot Position	Reaction Time (seconds)
Standing........................	Audible	Brake pedal	0.24
Standing........................	Bright light	Brake pedal	0.26
Standing........................	Stop light	Brake pedal	0.36
Standing........................	Audible	Accelerator	0.42
Standing........................	Bright light	Accelerator	0.44
Moving-normal road conditions......	Audible	Accelerator	0.46
Standing........................	Stop light	Accelerator	0.52
Moving-test conditions............	Stop light	Accelerator	0.68
Moving-normal road conditions......	Stop light	Accelerator	0.82*
Moving-test conditions............	None-stop light hidden	Accelerator	1.34
Moving-normal road conditions......	None-stop light hidden	Accelerator	1.65*

brake lights, can be found in (14). This experiment was executed like the eleven-car run on the GM test track, at about 40 miles per hour; suddenly the lead car braked, and the times when the sixth car braked and eleventh car braked were observed. These results are shown in Table 7. Here the brake reaction time was found to be about 1 second without brake lights and about $0.42 \sim 0.65$ seconds with brake lights, under the test conditions.

Furthermore, the relation of the reaction time with the sensitivity coefficient is mentioned in the following sources. T. Constantine and A. P. Young (17) suggested 1.0 second for reaction time in congested urban conditions after observation with the Kine method which used two movie cameras and obtained data from film analysis. They also showed that the reaction times which give the maximum correlation coefficient are almost the same in both theoretical formulae, i.e., Chandler et al. and Greenberg, except for deceleration.

From the above data, it can be concluded that the reaction time is $0.5 \sim 1.0$ second in congested urban conditions, and $1.5 \sim 2.0$ seconds in normal conditions without brake lights. When the brake lights are on, the reaction time decreases to $0.5 \sim 0.8$ seconds.

Speeds of 30 to 50 ft/sec are considered to be appropriate for the reciprocal spacing coefficient, c_2, under normal condition as shown in Table 5.

<u>Acceleration and Velocity Limits</u>. In the <u>Traffic Engineering Handbook</u> (12), Table 22, page 24, the speed-acceleration relationship is shown. This information is reproduced as Table 8.

Table 7. Brake Reaction Time, from (14)

Test Condition	A Response to Brake light only		B Response to any		C Brake light hidden except 1st car & 11th
Run number	t_6 (s)	t_{11} (s)	t_6 (s)	t_{11} (s)	t_{11} (s)
1	3.00	5.95	2.33	5.70	10.90
2	3.00	6.05	1.49	6.85	9.95
3	3.05	5.75	2.68	6.50	12.00
4	3.44	6.75	1.68	6.10	10.20
5	2.75	7.80	2.26	3.72	9.35
6	--	--	--	--	8.30
Ave/car	0.61	0.65	0.42	0.58	1.01

Table 8. Speed-Acceleration Relationship During Normal Acceleration from (12)

	Normal Acceleration in Miles per Hour per Second, According to Speed Reached in Miles per Hour								
	10	15	20	25	(mph) 30	35	40	45	50
Passenger cars:									
Rural*	6.0	5.7	5.3	4.9	4.5	4.3	4.1	3.7	3.5
Urban	3.6	3.3	3.0	2.5	2.1	---	---	---	---
Single-unit trucks	2.5	1.8	1.6	1.2	0.9	0.6	0.4	0.3	0.2
Semi-trailer units	1.8	1.3	0.9	0.7	0.6	0.4	0.3	0.3	0.2
Intercity busses	2.3	1.7	1.4	1.2	0.9	0.7	0.6	0.4	0.3

* Obtained while accelerating up to a running speed of 60 mph.

Obtained while accelerating up to a running speed of 35 mph.

Table 2.5 of (12) shows passenger car deceleration with the foot off the accelerator, with the clutch engaged, and without using the brakes as:

Speed of car [mph]	70	60	50	40	30	20
Acceleration [m/h·s]	2.5	2.1	1.7	1.4	1.1	0.8

Thus the equation for deceleration without utilizing brakes is:

$$\dot{v}[ft/s^2] = 0.035v \tag{3-9}$$

Desired Distance. W. Helly (16) applied the following equation to characterize the desired distance based on the data on page 63 of the Traffic Engineering Handbook 1950 (12),

$$D[ft] = 20 + 1\dot{X}_n(t)[ft/s]. \tag{3-10}$$

This equation seems to be appropriate; consequently, (3-10) is adopted as the desired distance equation in this research. Now let us use the same supposition which W. Helly (16) used to determine the value of c_5, the coefficient of desired distance control in equation (3-6).

Suppose a car is travelling alone at its maximum speed. It is assumed that the car in front of it is so far ahead that the c_5 term of of equation (3-6) contributes more to the desired acceleration than does the c_2 term. Then, the desired acceleration is positive, but, since the car is at its maximum velocity, the actual acceleration is zero. Now, suppose that the car overtakes another car whose velocity is zero. Equation (3-6) now becomes:

$$\ddot{X}_{n+1}(t+T) = \frac{-c_2\dot{X}_{n+1}(t)}{X_n(t)-X_{n+1}(t)} + c_5[X_n(t) - X_{n+1}(t) - D] \tag{3-11}$$

At the point where our driver first begins to react to the stationary vehicle, $\ddot{X}_{n+1}$ will change sign from positive to negative. Thus, the point where this occurs is characterized by

$$\frac{-c_2\dot{X}_{n+1}(t)}{X_n(t) - X_{n+1}(t)} + c_5\left[X_n(t) - X_{n+1}(t) - D\right] = 0$$

or, rearranging terms and substituting for D,

$$\frac{c_5}{c_2} = \frac{\dot{X}_{n+1}(t)}{\Delta XR[\Delta XR - \alpha - \beta\dot{X}_{n+1}(t)]} \tag{3-12}$$

where ΔXR is the distance at which a driver going at $\dot{X}_{n+1}(t)$ first decelerates when approaching a stationary obstacle.

The problem is to determine at what point the driver notices the obstacle. The answer can be found in a paper written by R. M. Michaels and L. W. Cozan (9). In their experiment, the range of angular velocity at the beginning of displacement, which is equivalent to the threshold of equation (2-31) in Chapter II, was 4 to 40 minutes of arc per second (0.00116 ~ 0.0116 radian/second). The value of this threshold is varied by visual and other conditions. Let us choose the midpoint of the above range, 22'/s (0.00638 radian/s), as the threshold for noticing the difference in velocity here; from this value let us calculate the relation between the speed of the car and the distance from which the driver notices the stationary car, the width of which is assumed to be six feet, at the distances of 100 feet, 200 feet, and 300 feet, respectively, by using equation (2-31). Next, let us calculate the value c_5/c_2 in equation (3-12) by substituting $D = 20 + \dot{X}_n(t)$ from equation (3-10).

Finally, we can obtain the value of c_5 by substituting c_2=31.5mph.

Table 9. Calculation of c_2 from Threshold

Given Distance ΔX_R [ft]	(a) Speed by (2 - 31) $\dot{X}_{n+1}(t)$ [ft/s]	(b) c_5/c_2 by (3 - 12) [1/ft·s]	c_5 when c_2 = 31.5 [1/s^2]	c_5 when c_2 = 50.1 [1/s^2]
100	10.6	0.00153	0.0482	0.0767
200	42.5	0.00155	0.0488	0.0777
300	95.7	0.00173	0.0515	0.0867

(a) Equation (2 - 31) is $\frac{d\theta}{dt} \cong W \left| \frac{\dot{X}n(t) - \dot{X}n_{+1}(t)}{Xn(t) - Xn_{+1}(t)} \right|$

where $\frac{d\theta}{dt}$ = 0.00638 radian/s, W= 6 ft, $\dot{X}n(t)$= 0, and $Xn(t) - Xn_{+1}(t) = \Delta X_R$.

(b) Equation (3 - 12) is $\frac{c_5}{c_2} = \frac{\dot{X}n_{+1}(t)}{\Delta X_R \; (\Delta X_R - \alpha - \beta Xn+1(t))}$

where α = 20, β = 1, c_2=31.5 or 50.1 ft/sec

All of the above process is shown in Table 9. c_5 is obtained as almost the same value, around 0.05 s^{-2}, which seems to suggest the correctness of the model and the appropriateness of the assumptions.

Finally, the simulation model equation (3-5) can be rewritten:

$$\ddot{X}_{n+1}(t+T) = \frac{c_2[\dot{X}_n(t) - \dot{X}_{n+1}(t)]}{[X_n(t) - X_{n+1}(t)]} + c_5[X_n(t) - X_{n+1}(t) - D] \quad (3\text{-}13)$$

where the parameters are as shown in Table 10.

Trial Runs and Modification

Some trial runs were executed to confirm the response of the simulation model under various conditions. In some specific runs, certain unrealistic behavior was observed: 1) a car, which was very close to the one immediately ahead and could have stopped if the driver had applied its maximum deceleration, collided with it. 2) a car, which could have stopped from considerably high speed in accordance with the behavior of the car ahead, crashed into it when it moved a little bit to adjust the distance to its desired distance.

The Greenberg model and its experimental data were observed from some specific car maneuvers and do not cover every condition. Thus, even though the simulation model was built based on the Greenberg model and data, it does not always work correctly. The cause of the above unrealistic behavior is that the headway in the Greenberg model is defined as the distance from the center of the car to the center of another car, or from front bumper to front bumper. A collision will occur when this headway drops below 15 feet; then the situation can occur in Greenberg's model when the driver does not apply the maximum deceleration even if the space between the cars is less than one foot. However, in actual

Table 10. Parameters of Simulation Model

Symbol	Description	Unit	Ordinary Range	Typical Value	Condition
c_2	Velocity control	ft/sec	31.5~50.1	40	
c_5	headway control	$1/sec^2$	0.05~0.08	0.065	
T	reaction time	sec	0.5~0.8	0.75	if BLT=1
T_2	reaction time		1.0~2.0	1.5	if BLT=0
MA	maximum acceleration $MA = (MS - \dot{X}_{n+1})$	ft/sec^2			
MS	maximum speed	ft/sec	100~200	140	
	Acceleration coefficient	1/sec		0.07	
MD	maximum deceleration	ft/sec^2	15~28	20	
D	desired headway $D=a+b\dot{X}_n$	ft			
a	minimum headway	ft		1.5	
b	desired headway coeff.	sec	1~1.5	1.2	
CDC	coasting deceleration $CDC = B\dot{X}_n$	ft/sec^2			
B	coasting deceleration coeff.	1/sec		0.035	
BLT	brake light BLT = 0 if X_n -CDC; 1 if X_n -CDC			1or 0	

practice most drivers seem to apply the maximum deceleration in this case and the headway which is always observed by drivers to control their cars is not the distance from the front bumper to the front bumper, but the distance from the front bumper to the rear bumper, particularly when the relative speed is negative.

It is generally recognized that driver sensitivity varies between the case of acceleration and deceleration, that drivers respond more quickly to deceleration than to acceleration.

The simulation model was modified to reflect the fact that the driver's response is not based on center-to-center distance but rather on the distance from the driver's front bumper to the rear bumper of the car in front of him when the relative speed is negative.

Acceleration Noise

Most mathematical models ignore the acceleration noise; however, as R. Herman and R. Rothery have reported (18) acceleration noise plays a significant role at low vehicle traveling speed.

R. Herman et al. (10) have experimented to find the standard deviation of the acceleration noise, σ_D, under typical driving conditions. Their results are:

(1) $\sigma_D = 0.01g = 0.32\ ft/s^2$ for a car proceeding alone on a highway. This value does not depend much on speed or the driver.

(2) $\sigma_D = 0.03g = 0.96\ ft/s^2$ for a car proceeding smoothly in moderate traffic at 35 miles per hour.

Then the acceleration noise, $\sigma_D = 0.32\ ft/s^2$, is added to each car's acceleration in the simulation model as follows,

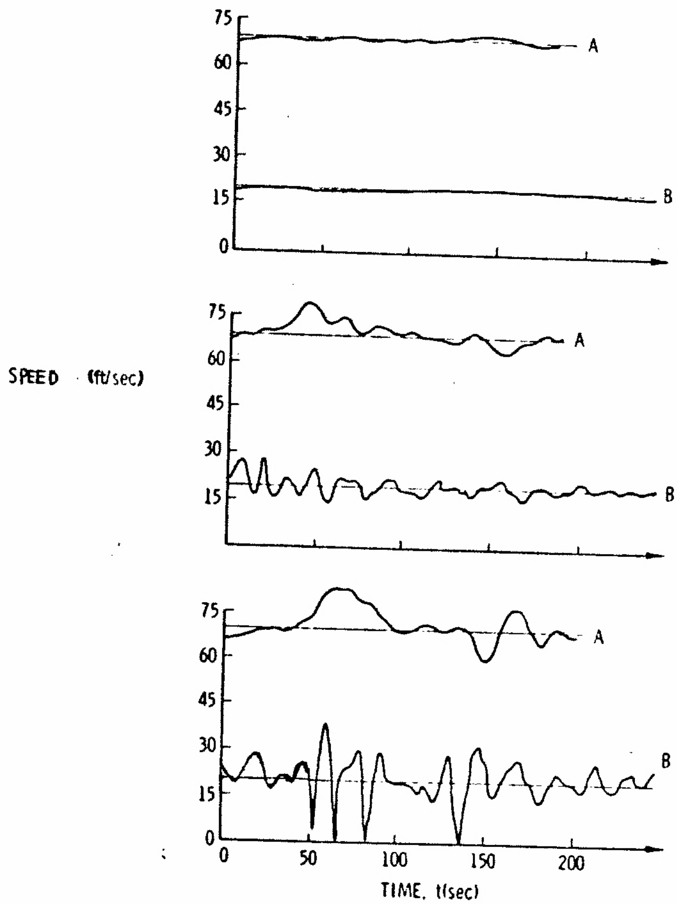
$$\ddot{X}'_{n+1}(t) = \ddot{X}_{n+1}(t) + N(0,\ 0.32\ ft/s^2), \qquad (3\text{-}14)$$

where $\ddot{X}'_{n+1}(t)$ is acceleration with noise, N(0, 0.32) is a normally distributed random variable with mean 0 and standard deviation 0.32.

The effect of the acceleration noise which is added to the simulation model is compared with the result of the experiment by R. Herman (18), by comparing Fibure 18 and RUNS-NS-1, 2, and 3 in Appendix C.

In RUN-NS-1 and RUN-NS-2, the initial speeds of all cars are equal and the initial headways of all cars are set at the desired distances; then oscillations of speed gradually increase. It is not desirable to start a simulation run with acceleration noise added with the above initial conditions because the effect of the acceleration noise will appear gradually, necessitating a wait until the influence of the acceleration noise seems to stabilize. However, it is also not desirable to start a simulation run with such extreme conditions that the effect of acceleration noise is too strong. Therefore, it was decided that the initial condition of the acceleration noise added simulations would be set at speeds and distances equal to the situation at 20 seconds of RUN-NS-1 except in special cases. RUN NS-3 shows the effect of the acceleration noise which starts with this initial condition.

Comparing these simulation runs with the results of experiments (18), it can be said that the simulation model exhibits almost the same response tendency as the result of that experiment concerned with acceleration noise, although in the case in which the first car's speed was 20 feet per second, the simulation run had a stronger response than the cited experiments. The reason for this difference may be that in actual-



Speed profiles recorded from two runs A and B for the three instrumented vehicles positioned as the first (top graph), middle (center graph) and last vehicle (bottom graph) of an eleven vehicle platoon.

Figure 18. Effect of Acceleration Noise from (18)

ity a driver observes and reacts not only to the car directly in front of him, but also to two or more cars in front, while in this simulation model the dirver is assumed to react only to the car immediately in front. Thus, the propagation of effect may absorb more in actual situations, to some degree, than in the case of this simulation model.

Characteristics of the Simulation Model

Now that the simulation model has been described in detail, let us revise it totally from the viewpoint of steady state and stability. Since the simulation model has been developed based on the Greenberg model, it has inherited the characteristics of the Greenberg model relative to the velocity control function as long as the difference in speed between cars exists, i.e., the relative speed is not zero. Another characteristic of the simulation model is the headway control function. This function may not be as strong as the velocity control function because its coefficient is comparatively small as long as a relative speed exists. However, when the steady state is assumed, this means that the relative speed is zero and no more reaction will occur. This situation will be achieved in the case of this simulation model only when all of the cars' headways have been adjusted to their desired distances at their traveling speed; i.e., the following equation must be satisfied:

$$AD_j = DD_j = a + b\dot{X}_j \tag{3-15}$$

where AD_j is the actual headway of car j [ft]

DD_j is the desired headway of car j [ft]

a is the minimum desired headway [ft]

b is the desired headway coefficient [s]

$\dot{X}_j$ is the speed of car j. [ft/s]

Equation (3-15) can be rewritten as

$$u = c_1(s - s_j) \tag{3-16}$$

where $u = \dot{X}_j$

$c_1 = 1/b$

$s = D_j$: Headway

$s_j = a$

Also, we can rewrite (3-16) by multiplying k, concentration, to both sides of the equation and using the relation $s = 1/k$, $s_j = 1/k_j$, as

$$uk = kc_1(s - s_j) = c_1(1 - k/k_j). \tag{3-17}$$

In steady state $q = uk$, then equation (3-17) is the same as equation (2-10), which is the steady state of the Chandler et al. model.

By this headway control function, the combination of the initial speed and distance is not dominant in this simulation model. In other words, the steady state of the simulation does not depend on the initial headway and speed, while in the Greenberg and other mathematical models which were mentioned previously, the combination of initial velocity and headway dominates the steady state.

Next let us consider stability. In this simulation model, it is impossible to find a simple relationship like that in the Chandler et al. model, because the relation is no longer linear and various other factors are involved. In this complex model, the meaning of stability is not so clear and important as before. For example, it is assumed that a small braking wave increases in propagation. This is an unstable situation; however, after the cars have stopped, this situation could be called stable.

CHAPTER IV

APPLICATION OF THE SIMULATION MODEL

Emergency Stop and Rear End Collision

Emergency stop behavior was chosen as the first example of the application of the simulation model.

Design of Experiment

Since this simulation model is not linear and includes many factors, it is not easy to ascertain the influence of each factor upon stability. We cannot conclude that the system is unstable like the previous, simpler mathematical models even if the amplitude becomes large because of the car performance limits.

Therefore, it would be better to measure stability by the occurence of collisions or the degree of collision damage.

The First Vehicle Maneuver. Under conditions of ordinary vehicle travel a collision will rarely happen. It is assumed in this application that the first car decelerates from a specific speed to a complete stop by 10 ft/s^2. The 10 ft/s^2 deceleration is such a strong degree of braking that the occupants of the vehicle will feel uncomfortable. Practical values of deceleration used in everyday traffic conditions rarely exceed 8 to 9 ft/s^2 (12). Then this situation may be considerably severe.

Factors. The factors which seem to be related to stability are velocity control, c_2, headway control, c_5, reaction time when the preceding car's brake lights are off, T_1 and on T_2, maximum acceleration, MA (or maximum speed, MS), maximum deceleration, MD, desired distance, D (or its coefficient, b), and initial speed, IS. But in the case of emergency stop, T_1 and MA(MS) are not concerned and are deleted.

Combinations. The six relevant factors are too many to deal with in a complete factorial experiment (19); even if two levels are chosen for each factor and a fractional experiment is tried, $2^6 = 64$ runs are necessary. Consequently a 1/4 replicate of a 2^6 factorial experiment has been chosen to analyze this problem. The two levels of each factor are determined as values of almost ± 25 percent from its typical value (these points might be almost 2σ points). The factors have been assigned as shown in Table 11 by assuming that the interactions of three-factor combinations are not significant. The emergency stop experimental design simulation run has been performed both with and without acceleration noise.

Results of Simulation

Results of the simulation are summarized in Tables 12 and 13, and some typical runs are shown in Appendix C.

As shown in (20), the degree of damage in accidents has a linear relation to impact speed. Therefore, analysis of variance (ANOVA) (19) has been executed on both the number of collided vehicles and total impact speed. This is a statistical method for analyzing the effects of the various factors. The ANOVA results are shown in Tables 14 - 16. The effect of c_2(-effect, i.e., higher value of c_2 will reduce total impact speed or the number of collisions), T_2 (+ effect), b (-effect), and IS (+ effect) commonly appear as significant in all these analyses. In addition, the interactions $c_2 \times T_2$ and IS x MD (+ effects) become significant in the case of number of collisions. The significant difference between situations with and without acceleration cannot be seen in this type of maneuver. It is notable that the effect of maximum deceleration is not as significant as other factors. This suggests that the improvement of maximum deceleration or differences in maximum deceleration in normal dry road surface conditions are not so effective as increasing velocity control, i.e., responding strongly to relative speed, decreasing reaction time, increasing headway, or decreasing traveling speed in the case of emergency stop.

The effects of the significant factors are shown in Figure 19. To observe the effects of these factors further, the standard run in which all factors are set at their typical value and the runs in which each factor individually moves $\pm$ 25 percent from its typical value have been executed. The results of these runs are shown in Table 18 and Figure 20.

Stop-Run-Stop Maneuver

The first application of the emergency stop maneuver did not deal with the acceleration and the reaction time T_1 when the brake lights are off. Therefore, as the second application the stop-run-stop maneuver was chosen. We often experience this type of maneuver when we drive into congested traffic or out from it.

Design of Experiment

First Vehicle Maneuver. The first vehicle maneuver involves starting by an acceleration of 6 ft/s^2 for two seconds from a station-

Table 11. Combinations of Factors for Design of Experiment in Emergency Stop

Factor / Run	c_2	c_5	T_2	b	IS	MD	(ID)
1	30	.05	.5	.8	45	-15	56
2	50	.05	.5	.8	75	-25	80
3	30	.08	.5	.8	75	-15	80
4	50	.08	.5	.8	45	-25	56
5	30	.05	1.0	.8	45	-25	56
6	50	.05	1.0	.8	75	-15	80
7	30	.08	1.0	.8	75	-25	80
8	50	.08	1.0	.8	45	-15	56
9	30	.05	.5	1.5	75	-25	132.5
10	50	.05	.5	1.5	45	-15	87.5
11	30	.08	.5	1.5	45	-25	87.5
12	50	.08	.5	1.5	75	-15	132.5
13	30	.05	1.0	1.5	75	-15	132.5
14	50	.05	1.0	1.5	45	-25	87.5
15	30	.08	1.0	1.5	45	-15	87.5
16	50	.08	1.0	1.5	75	-25	132.5

where c_2 is velocity control sensitivity ft/s

c_5 is headway control sensitivity $1/s^2$

T_2 is reaction time when brake light is on s

b is coefficient of desired headway s

IS is initial speed ft/s

MD is maximum deceleration ft/s^2

ID is initial distance not an actual factor but determined as ID = 20 + b · IS

Table 12. Summary of Emergency Stop Without Acceleration Noise
Collision Time (s) and Collision Speed (ft/s)

Run	Crash	Car #1	#2	#3	#4	#5	#6	#7	#8	#9	number of crashes total speed
1	Time s Speed ft/s		6.25 3.4	6.75 10.6	7.5 14.1	8.5 18.1	9.5 18.9	10.5 18.7	11.5 18.4	12.5 18.2	8 120.4
2	Time Speed					10.0 6.6	10.5 17.1	11.25 21.7	12.0 26.5	12.75 32.4	5 104.3
3	Time Speed		7.5 30.4	8.0 42.0	8.75 51.3	9.5 60.5	10.5 66.1	11.5 69.4	12.5 71.9	13.5 74.0	8 465.6
4	Time Speed										0 0
5	Time Speed		5.25 31.8	6.25 41.1	7.25 43.5	8.25 44.5	9.25 44.9	10.25 50.0	11.25 45.0	12.25 45.0	8 345.8
6	Time Speed		7.0 36.3	7.75 49.9	8.25 6.06	9.25 68.6	10.25 72.2	11.25 74.0	12.25 74.8	13.25 75.0	8 511.4
7	Time Speed		6.5 51.2	7.25 68.3	8.25 73.1	9.25 74.6	10.25 72.2	11.25 74.0	12.25 74.8	13.25 75.0	8 511.4
8	Time Speed		5.5 21.1	6.25 28.2	7.0 37.1	8.0 42.1	9.0 43.6	10.0 44.4	11.0 44.9	12.0 45.0	8 306.4
9	Time Speed			10.5 24.1	11.75 42.8	13.25 54.3	14.75 62.2	16.25 67.2	17.75 70.5	19.5 67.9	7 389.0
10	Time Speed										0 0
11	Time Speed										0 0
12	Time Speed										0 0
13	Time Speed	8.0 20.4	8.75 42.1	10.0 59.3	11.50 67.8	13.00 68.9	14.75 70.2	16.5 70.6	18.25 70.8	20.0 70.9	9 541.0
14	Time Speed										0 0

Table 12. (Cont'd.) Summary of Emergency Stop Without Acceleration Noise

Collision Time (s) and Collision Speed (ft/s)

Run	Crash	Car #1	#2	#3	#4	#5	#6	#7	#8	#9	number of crashes Total Speed
15	Time Speed		7.5 12.6	8.75 21.9	10.25 29.0	11.75 34.7	13.5 34.7	15.25 35.8	17.0 36.8	18.75 37.5	8 243.0
16	Time Speed			9.75 20.4	11.25 33.5	12.75 43.2	14.25 52.5	15.75 59.5	17.25 62.5	19.00 61.7	7 333.3

Table 13. Summary of Emergency Stop with Acceleration Noise
Collision Time (s) and Collision Speed (ft/s)

Run	Crash	Car #1	#2	#3	#4	#5	#6	#7	#8	#9	number of crashes / Total Speed
1	Time		6.25	7.0	7.75	8.75	9.75	11.0	12.75		7
	Speed		4.2	8.0	12.4	12.6	13.8	11.6	8.0		70.6
2	Time					10.0	10.75	11.75	12.75	13.75	5
	Speed					8.9	15.2	16.1	15.5	14.1	69.8
3	Time		7.5	8.0	8.75	9.5	10.5	11.5	12.5	14.25	8
	Speed		30.8	42.9	52.3	60.8	62.5	60.5	57.3	59.7	426.8
4	Time										0
	Speed										0
5	Time		5.5	6.25	7.25	8.25	9.5	10.75	12.25	13.75	8
	Speed		26.6	39.4	42.3	44.7	40.5	39.5	30.6	25.4	289.0
6	Time		7.0	7.5	8.25	9.0	10.0	11.25	12.50	13.75	8
	Speed		37.0	50.8	60.3	68.6	74.0	74.5	74.5	68.7	508.4
7	Time		6.25	7.25	8.25	9.25	10.25	11.0	12.0	12.75	8
	Speed		51.5	67.9	73.5	76.3	76.5	72.1	66.3	63.5	547.6
8	Time		5.75	6.25	7.25	8.25	9.25	10.5	12.75		7
	Speed		18.0	25.1	27.6	27.3	30.8	38.6	32.7		200.1
9	Time			10.5	12.0	13.5	15.25	17.0	18.5	20.0	7
	Speed			24.3	29.4	33.7	30.4	26.7	29.1	29.7	203.3
10	Time										0
	Speed										0
11	Time										0
	Speed										0
12	Time										0
	Speed										0
13	Time	8.0	8.75	10.0	11.5	13.25	15.0	16.75	18.5	20.0	9
	Speed	20.5	42.3	56.9	62.9	61.8	61.8	59.9	57.8	59.7	483.6
14	Time										
	Speed										0
15	Time		7.5	9.0	10.5	12.0	13.75	15.50	17.25	19.25	8
	Speed		12.9	17.5	22.4	26.6	27.5	28.9	30.3	26.5	192.6
16	Time			10.0	11.25	12.75	14.25	16.0	17.75	19.5	7
	Speed			21.1	32.6	37.8	44.2	44.4	46.7	48.4	275.2

Table 14. ANOVA for Collision Speed in Emergency Stop without Acceleration Noise

($\overline{X}$ = 245.5 ft/s)

Source	Effect	df	SS	% of SS	MS/Total SS
c_2	-177.1	1	125422.22	18.20	18.20**
c_5	-12.1	1	580.81	0.08	0.08
T_2	221.1	1	195540.84	28.38	28.38**
b	-114.4	1	52303.69	7.59	7.59*
c_2xIS,$c_5$xb	-76.4	1	23332.56	3.39	3.39
IS	237.0	1	224723.40	32.62	32.62**
MD	-56.0	1	12555.20	1.82	1.82
Error	---	8	54514.14	7.91	.989
Total	---	15	688972.86	100.0	---

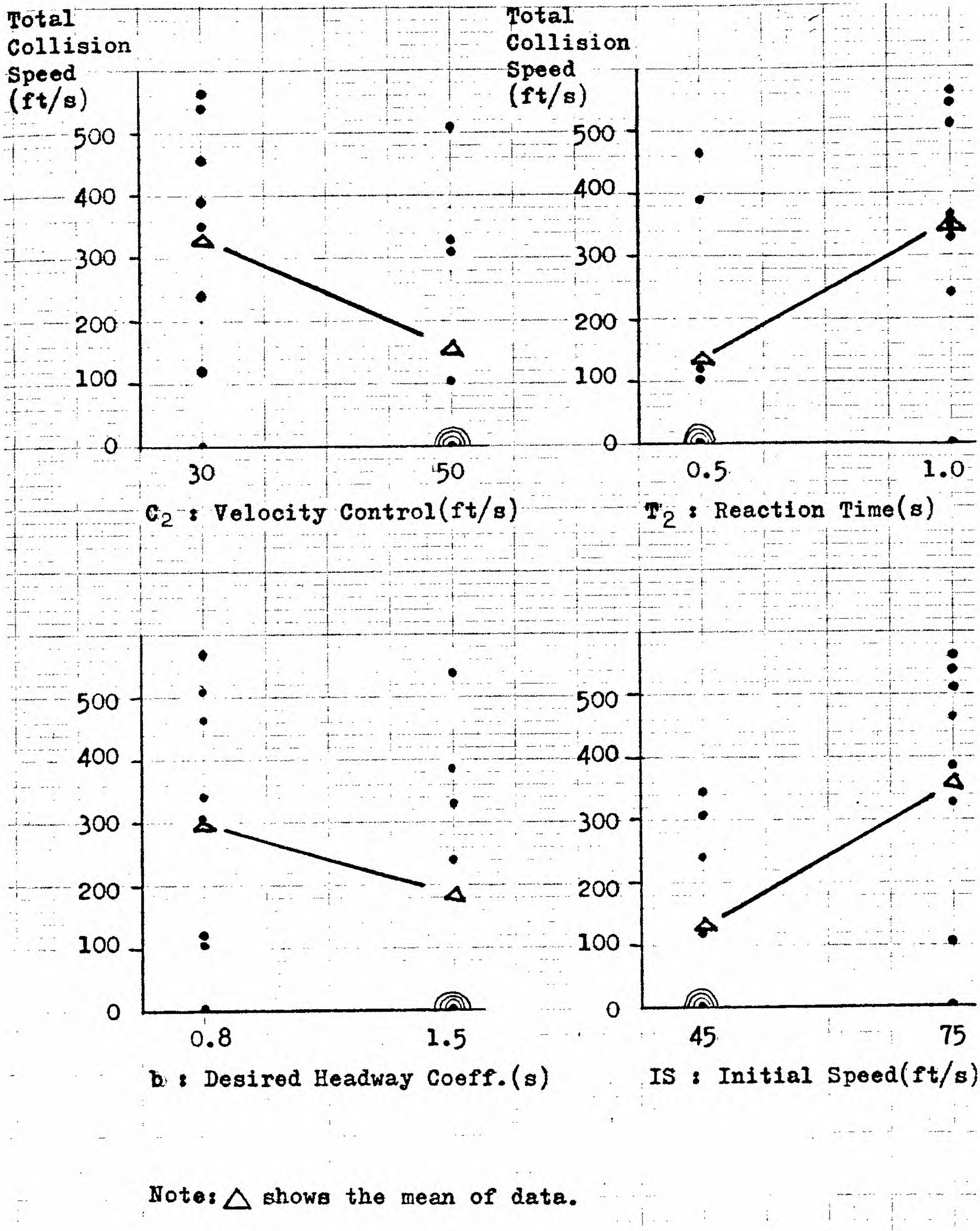


Figure 19. Effect of Factors in Emergency Stop Without Acceleration

Table 15. ANOVA for Number of Collisions in Emergency Stop without Acceleration Noise

($\bar{X}$ = 5.25 vehicle)

Source	Effect	df	SS	% of SS	MS/Total SS
c_2	-3.5	1	49.0	23.22	23.22**
c_5	-0.75	1	2.25	1.07	1.07
ISxb, c_2xc_5	1.25	1	6.25	2.96	2.96
T_2	3.5	1	49.0	23.22	23.22**
c_5xT_2, ISxMD	2.25	1	20.25	9.60	9.60*
b	-2.75	1	30.25	14.34	14.34**
IS	2.5	1	25.0	11.85	11.85*
MD	-1.75	1	12.25	5.81	5.81
Error	---	7	16.75	7.95	1.136
Total	---	15	211.0	100.0	---

$F^{1}_{7\ 0.05} = 5.59 \quad F^{1}_{7\ 0.02} = 12.2$

Table 16. ANOVA for Collision Speed in Emergency Stop with Acceleration Noise

($\bar{X}$ = 204.2 ft/s)

Source	Effect	df	SS	% of SS	MS/Total SS
c_2	-145.0	1	84100.00	14.14	14.14**
c_5	2.2	1	19.36	0.003	0.003
T_2	215.8	1	186192.25	31.30	31.30**
b	-119.7	1	57312.36	9.63	9.63*
c_2xIS, $c_5$xb	-55.7	1	12984.60	2.18	2.18
IS	220.3	1	194128.36	32.63	32.63**
MD	-62.2	1	15450.49	2.60	2.60
Error	---	8	44673.62	7.51	0.939
Total	---	15	594861.04	100.0	---

Table 17. ANOVA for Number of Collisions in Emergency Stop with Acceleration Noise

($\bar{X}$ = 5.13 vehicle)

Source	Effect	df	SS	% of SS	MS/Total SS
c_2	-3.5	1	49.0	24.29	24.29**
c_5	-0.75	1	2.25	1.12	1.12
ISxb, $c_2 x c_5$	1.0	1	4.0	1.98	1.98
T_2	3.5	1	49.0	24.29	24.29**
$c_5 x T_2$, ISxMD	2.0	1	16.0	7.93	7.93*
b	-2.5	1	25.0	12.39	12.39*
IS	2.75	1	30.25	14.99	14.99**
MD	-1.5	1	9.0	4.46	4.46
Error	---	7	17.25	8.55	1.221
Total	---	15	201.75	100.0	---

Table 18. Collision Speed at Three Levels in Emergency Stop

Run	Crash	Car #1	#2	#3	#4	#5	#6	#7	#8	#9	number of crashes / Total crash speed
1	Time			8.5	9.5	10.5	12.0	13.25	14.75	16.25	7
	Speed			16.3	24.0	26.4	30.3	34.7	32.9	30.7	195.3
2	Time		7.5	8.5	9.75	11.0	12.25	13.75	15.25	16.50	8
	Speed		21.0	28.6	35.5	58.4	43.8	39.5	36.3	37.6	300.7
3	Time					11.25	12.25				2
	Speed					.5	7.1				7.6
4	Time			7.25	8.0	8.75	9.75	11.0	12.25	13.5	7
	Speed			23.9	32.1	40.2	46.0	45.3	43.9	40.7	272.1
5	Time										0
	Speed										0
6	Time										0
	Speed										0
7	Time			9.25	10.25	11.25	12.5	13.75	15.25	16.5	7
	Speed			28.8	40.0	50.4	54.9	57.8	53.0	53.8	338.7
8	Time		8.0	8.75	9.75	11.0	12.25	13.5	15.0	16.5	8
	Speed		9.8	19.7	26.5	28.8	32.5	36.7	36.6	35.6	226.2
9	Time			8.5	9.5	10.75	12.0	13.25	14.75	16.25	7
	Speed			10.7	19.2	21.1	24.9	29.6	26.9	23.8	156.2
10	Time										0
	Speed										0
11	Time		7.25	8.25	9.5	10.75	12.0	13.5	15.0	16.5	8
	Speed		19.5	30.7	38.1	43.9	49.6	48.8	49.0	47.7	327.3

Note Run 1: Standard Condition: $c_2 = 40$, $b = 1.15$, $IS = 60$, $MD = -20$, $T_2 = 0.75$

Run 2: $c_2 = 30$ ft/s

Run 3: $c_2 = 50$ ft/s

Run 4: $b = 0.8$ s

Run 5: $b = 1.5$ s

Run 6: $IS = 45$ ft/s

Run 7: $IS = 75$ ft/s

Run 8: $MD = -15$ ft/s^2

Run 9: $MD = -25$ ft/s^2

Run 10: $T_2 = 0.5$ s

Run 11: $T_2 = 1.0$ s

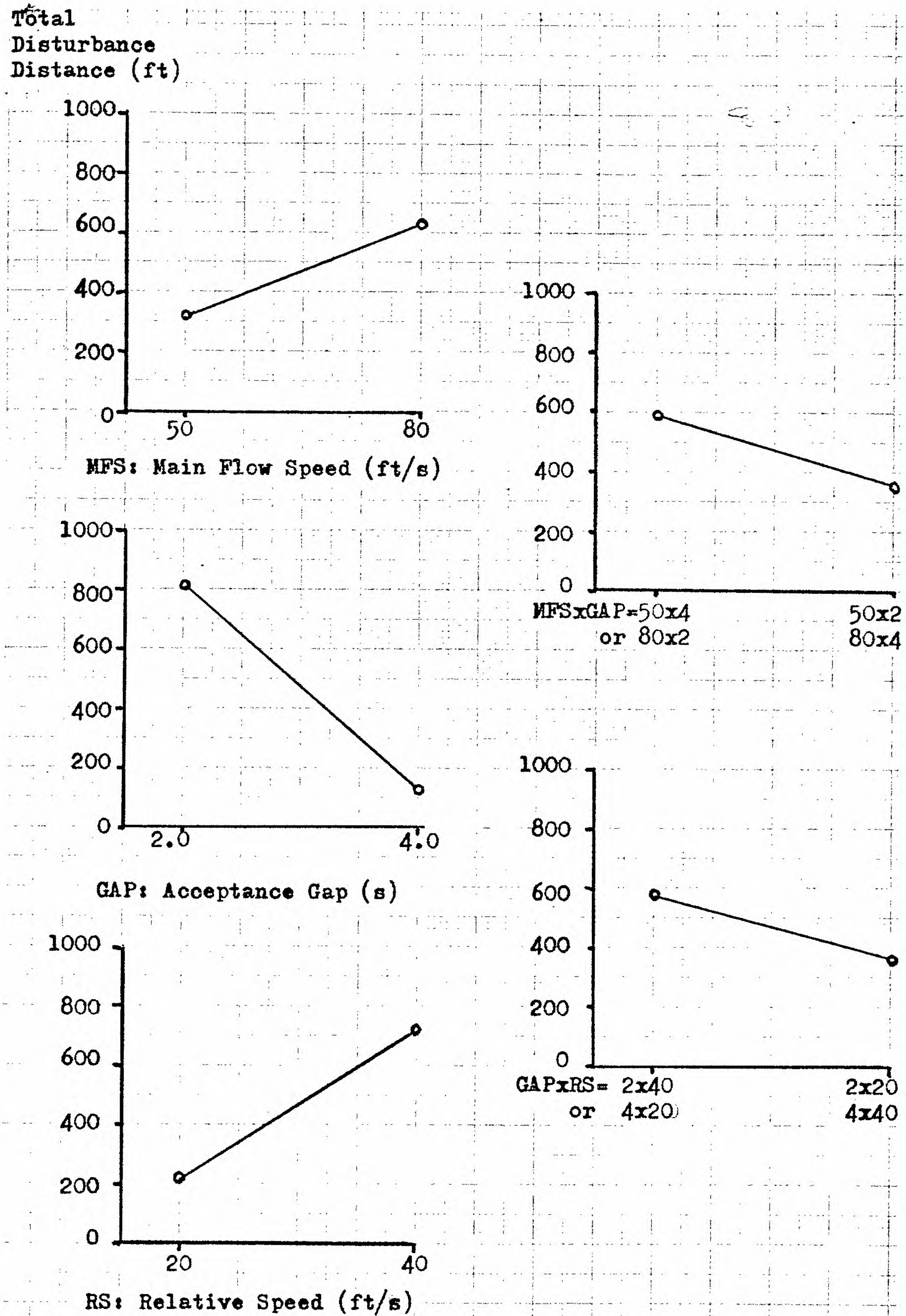


Figure 24. Effect of Factors on Total Disturbance Distance

ary position and braking by a deceleration of 6 ft/s^2 for two seconds to a complete stop. This maneuver may be considered as rather violent driver behavior.

Factors. As the factors, velocity control, c_2, headway control c_5, reaction time, T_1, T_2, the maximum acceleration, MA(which is determined by the maximum speed MS), the maximum deceleration, MD, the desired distance coefficient, b, and the initial speed, IS, were chosen.

Combinations. In a manner similar to the stop case, a 1/8 replicate of a 2^7 factorial experiment was applied assuming as insignificant interactions of more than three factors, Also, levels of factors were determined in the same manner as in the emergency stop case, i.e., values deviating approximately $\pm$ 25 percent from typical values. In Table 19 the combination of factors for this experiment are shown. Cases both with and without acceleration noise were executed.

Results of Simulation

Collision. Some results of the simulation are shown as RUN-SRS in Appencix C and summarized in Tables 20 and 21. This experiment is not a good one for analyzing collisions because only a few collisions occurred. In the case without acceleration noise, only $T_1(-)$, $T_2(+)$ and the confounded interactions $T_1 \times T_2$, MS x c_5, MD x $c_2(-)$ appeared as significant to both total collision speed and number of collided vehicles. Among the above confounded interactions, it seems that only $T_1 \times T_2$ is significant because the main effects of the others, i.e., MS, c_5, MD and c_2, are not significant. In the case with acceleration noise, the error term decreased slightly. The reason for this decrease is not clear, but it may be due to the initial conditions of this experiment: all cars are stationary with minimum headway, i.e., 20 feet. Then the introduction of acceleration noise may lend to a cancelling of certain nonlinear feedback loops, producing a more nearly linear model with resultant smaller residual error. Besides the above factors, the effect of MS(+) and the confounded interactions c_2 x T_1, b x MD, T_2 x MS(-) become significant for the number of collided vehicles. Furthermore, in addition to the above c_5 and the confounded interactions T_1 x MS, c_5 x T_2, c_2 x b(-) become significant for total collision speed although the ratio of SS (Sum of Squares) of each source above does not seem to be very different from the case without acceleration noise. The result of ANOVA seems to suggest that in this experiment, many factors influence the total collision speed and the number of collisions, interacting intricately; that T_1, T_2 and T_1 x T_2 are surely significant; and that c_2, i.e., velocity control, is not so influential as other factors here.

Wave Propagation and Amplitude. To observe the wave propagation, the time and location of three important points, i.e., the starting point, the peak of speed, and the stop, have been measured for the fifth car (#4 car) and the tenth car (#9 car); the result of the measurement is shown in Table 26. The wave propagation of the speed of the above three points is observed as shown in Figure 21.

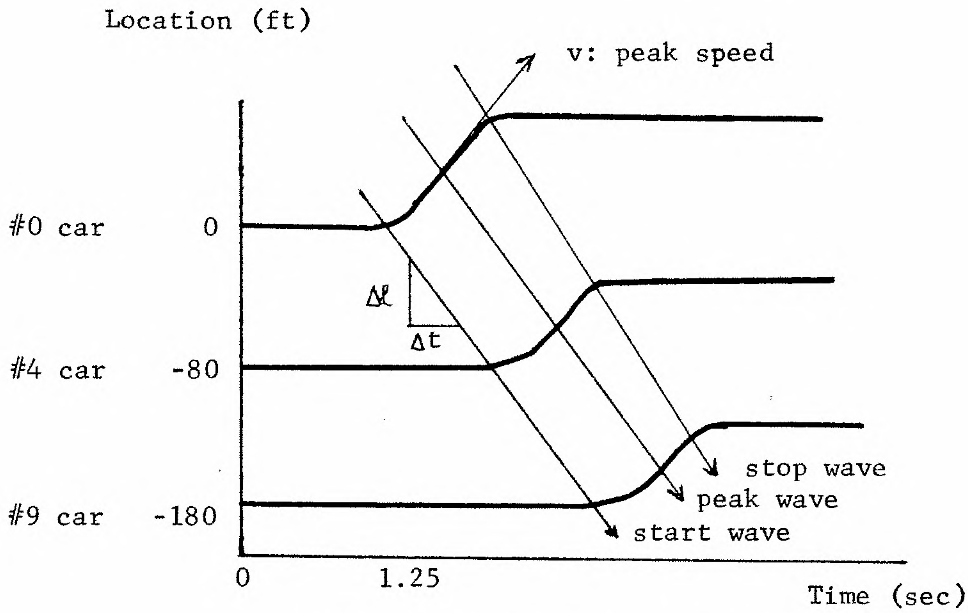


Figure 21. Wave Propagation

The first #0 vehicle started from the location point zero at the 1.25 second, reached its peak speed, 12.0 ft/s, on the location 10.5 feet at time 3.0 seconds and stopped on the location 24.0 feet at time 5.0 seconds. Thus, we can calculate the average wave speed as follows by using the time and location of the fifth or tenth vehicle:

$$SW = \frac{\Delta \ell}{\Delta t}$$

where SW is the average speed of wave of a specific maneuver.

Δt is the time difference from the time the first vehicle performed that specific maneuver to the time the fifth or the tenth vehicle executed it.

$\Delta \ell$ is the distance in which the meaeuvers of both the first and the fifth or tenth vehicle occurred.

The results of ANOVA for wave propagation are shown in Table 27, Table 28 and Table 29. In all three ANOVA's, the sum of the squares of such factors as c_5, b, MS, and their interactions, which are very small, are pooled into the error term finally. In the case of the start point and peak of speed point propagation, only $T_1(-)$, $T_2(-)$ and $T_1 \times T_2(-)$ become significant, and the difference in propagation between observation of the fifth car (#4) and the tenth car (#9) seems to be that the influence of T_2 slightly increases in the case of observation of the tenth car. In the case of wave propagation of the stopping point, besides the above factors, MD(+) becomes significant in observation of the fifth car, and $c_2(+)$ and MD(+) become significant in observation of the tenth car.

In this model, wave propagation is dominated by the reaction time, T_1 and T_2, and the distance between each pair of vehicles. However, the distance is usually not constant, but a function of such other factors as c_2, c_5, MD, etc.

According to observations in the Holland Tunnel (21), the average wave speed was 10.7 mph (15.7 ft/sec). This value is almost the same as the wave speed of the start and the peak of speed.

The amplitudes of speed were calculated as shown in Table 30. The result of ANOVA is shown in Table 31. It is interesting that the significant factors in the observation of the #4 car are different from those of the #9 car. However, the reason for this difference is not clear.

Through both the analyses dealing with collisions and amplitude, it is clearly shown that the reaction time T_1 and T_2, is the most dom-

Table 19. Combination of Factors for Design of Experiment for Stop-Run-Stop

Run	c_2	c_5	T_1	T_2	b	MD	MS
1	30	.05	1.0	.5	.8	15	105
2	50	.05	1.0	.5	1.5	25	105
3	30	.08	1.0	.5	1.5	15	175
4	50	.08	1.0	.5	.8	25	175
5	30	.05	2.0	.5	.8	25	175
6	50	.05	2.0	.5	1.5	15	175
7	30	.08	2.0	.5	1.5	25	105
8	50	.08	2.0	.5	.8	15	105
9	30	.05	1.0	1.0	1.5	25	175
10	50	.05	1.0	1.0	.8	15	175
11	30	.08	1.0	1.0	.8	25	105
12	50	.08	1.0	1.0	1.5	15	105
13	30	.05	2.0	1.0	1.5	15	105
14	50	.05	2.0	1.0	.8	25	105
15	30	.08	2.0	1.0	.8	15	175
16	50	.08	2.0	1.0	1.5	25	175

where c_2 is velocity control sensitivity ft/s

c_5 is headway control sensitivity $1/s^2$

T_1 is reaction time when brake light is off s

T_2 is reaction time when brake light is on s

b is coefficient of desired headway s

MD is maximum deceleration ft/s^2

MS is maximum speed ft/s

Table 20. Summary of Stop-Run-Stop without Acceleration Noise
(Collision Time (s) and Collision Speed (ft/s))

Run	Crash	Car #1	#2	#3	#4	#5	#6	#7	#8	#9	number of crashes Total
1	Time										0
	Speed										0
2	Tim e										0
	Speed										0
3	Time										0
	Speed										0
4	Time										0
	Speed										0
5	Time										0
	Speed										0
6	Time										0
	Speed										0
7	Time										0
	Speed										0
8	Time										0
	Speed										0
9	Time		6.75	7.75	9.25	10.50	12.0	13.25	14.75	16.00	8
	Speed		2.5	5.7	9.8	13.3	9.7	13.9	10.4	14.3	79.6
10	Time	5.0	6.0	7.25	8.50	9.75	11.0	12.25	13.50	14.75	9
	Speed	9.9	18.0	21.5	23.9	26.0	28.0	29.9	31.8	33.6	222.6
11	Time										
	Speed										
12	Time				9.50	10.75	12.0	13.25	14.50	15.75	6
	Speed				2.6	3.9	5.3	6.7	8.2	9.6	36.3
13	Time										0
	Speed										0
14	Time										0
	Speed										0
15	Time		8.25								1
	Speed		5.8								5.8

Table 20 (Continued). Summary of Stop-Run-Stop without Acceleration Noise (Collision Time (s) and Collision Speed (ft/s))

Run	Crash	Car #1	#2	#3	#4	#5	#6	#7	#8	#9	number of crashes Total
16	Time									21.25	1
	Speed									9.8	9.8

Table 21. Summary of Stop-Run-Stop with Acceleration Noise (Collision Time (s) and Collision Speed (ft/s))

Run	Crash	Car #1	#2	#3	#4	#5	#6	#7	#8	#9	number of crashes Total Speed
1	Time										0
	Speed										0
2	Time										0
	Speed										0
3	Time										0
	Speed										0
4	Time										0
	Speed										0
5	Time										0
	Speed										0
6	Time										0
	Speed										0
7	Time										0
	Speed										0
8	Time										0
	Speed										0
9	Time		6.75	7.75	9.25	10.05	12.0	13.25	14.50	15.75	8
	Speed		1.1	10.2	7.3	12.0	15.4	11.4	15.2	17.6	90.2
10	Time	5.0	6.0	7.25	8.50	11.0	11.0	12.25	13.5	14.5	9
	Speed	9.0	17.1	21.0	24.3	29.5	29.5	31.8	33.8	39.9	233.4
11	Time									16.25	1
	Speed									4.5	4.5
12	Time				9.5	10.5	11.5	12.75	14.0	15.0	6
	Speed				1.9	6.9	12.0				67.0
13	Time										0
	Speed										0
14	Time										0
	Speed										0

Table 21. (Continued) Summary of Stop-Run-Stop with Acceleration Noise (Collision Time (s) and Collision Speed (ft/s))

Run	Crash	Car #1	#2	#3	#4	#5	#6	#7	#8	#9	number of crashes / Total Speed
15	Time										0
	Speed										0
16	Time									23.5	1
	Speed									5.6	5.6

Table 22. ANOVA for Collision Speed in Stop-Run-Stop without Acceleration Noise

($\bar{X}$ = 22.13 ft/s)

Source	Effect	SS	%ofSS	△ Terms are pooled into error term	df	MS/Total SS
c_2	22.91	2099.93	4.24		1	4.24
c_5	-30.29	3915.63	7.91		1	7.91
T_1	-40.36	6516.53	13.17		1	13.17*
c_2xT_1, T_2xMD, bxMS	-21.91	1920.63	3.88		1	3.88
c_5xT_1, bxMD, T_2xMS	35.19	4952.64	10.01		1	10.01
T_2	44.26	7836.68	15.83		1	15.83*
T_1xMD, c_5xb, c_2xT_2	22.91	2099.93	4.24		1	4.24
T_1xMS, c_5xT_2, c_2xb	-30.29	3915.63	7.91		1	7.91
b	-12.84	659.21	1.33△		-	---
T_1xT_2, $MSxc_5$, $MDxc_2$	-40.36	6516.53	13.17		1	13.17*
MD	-21.91	1920.63	3.88		1	3.88
MS	35.19	4952.64	10.01		1	10.01
Error	---	2191.03	4.43	5.76	4	1.44
Total	---	49497.64	100.0	--	15	---

$F_4(0.05) = 7.71$, $F_4(0.01) = 21.2$

Table 23. ANOVA for Number of Collisions in Stop-Run-Stop without Acceleration Noise

($\bar{\bar{X}}$ = 1.563 vehicle)

Source	Effect	SS	%of SS		df	MS/ Total SS
c_2	0.875	3.06	2.13	Δ terms are pooled into error term	1	2.13
c_5	-1.125	5.06	3.52		1	3.52
T_1	-2.625	27.56	19.15		1	19.15*
$c_2xT_1, T_2xMD, bxMS$	-0.875	3.06	2.13		1	2.13
c_5xT_1 $bxMD, T_2xMS$	1.625	10.56	7.34		1	7.34
T_2	3.125	39.06	27.14		1	27.14**
T_1xMD, c_5xb, c_2xT_2	0.875	3.06	2.13		1	2.13
T_1xMS, c_5xT_2, c_2xb	-1.125	5.06	3.52		1	3.52
b	0.625	1.56	1.09 Δ		-	---
$T_1xT_2, MSxc_5, MDxc_2$	-2.625	27.56	19.15		1	19.15*
MD	-0.875	3.06	2.13		1	2.13
MS	1.625	10.56	7.34		1	7.34
Error	---	4.68	3.25	4.34	4	1.09
Total	---	143.90	100.0	--	15	---

Table 24. ANOVA for Collision Speed in Stop-Run-Stop with Acceleration Noise

($\bar{X}$ = 25.04 ft/s)

Source	Effect	SS	%ofSS		df	MS/ Total SS
c_2	26.41	2790.48	4.89		1	4.89
c_5	-30.81	3797.64	6.65	△ terms are pooled into error term	1	6.65*
T_1	-48.69	9481.89	16.60		1	16.60**
c_2xT_1, T_2xMD, bxMS	-25.01	2502.50	4.38		1	4.38
c_5xT_1, bxMD, T_2xMS	32.21	4150.58	7.27		1	7.27*
T_2	50.09	10035.03	17.57		1	17.57**
T_1xMD, c_5xb, c_2xT_2	26.40	2790.48	4.89		1	4.89
T_1xMS, c_5xT_2, c_2xb	-30.81	3797.64	6.65		1	6.65*
b	-9.31	352.50	.62△		-	
T_1xT_2, $MSxc_5$, $MDxc_2$	-48.69	9481.89	16.60		1	16.60**
MD	-25.01	2502.50	4.38		1	4.38
MS	32.21	4150.58	7.27		1	7.27*
Error	---	1283.46	2.24	2.86	4	.715
Total	---	57117.18	100.0		15	---

$F_{1.4}(0.05) = 7.71$ $F_{1.4}(0.01) = 21.2$

Table 25. ANOVA for Number of Collisions in Stop-Run-Stop with Acceleration Noise

($\bar{X}$ = 1.56 vehicle)

Source	Effect	SS	%ofSS		df	MS/ Total SS
c_2	.88	3.06	2.13	△ terms are pooled into error term	1	2.13
c_5	-1.13	5.06	3.52		1	3.52
T_1	-2.88	33.06	22.97		1	22.97**
c_2xT_1,T_2xMD,bxMS	-.63	1.56	1.09 △		-	
c_5xT_1,bxMD,T_2x	1.38	7.56	5.25		1	5.25*
T_2	3.13	39.06	27.14		1	27.14**
T_1xMD,$c_5$xb,c_2xT_2	.88	3.06	2.13		1	2.13
T_1xMS,c_5xT_2,$c_2$xb	-1.13	5.06	3.52		1	3.52
b	.63	1.56	1.09 △		-	
T_1xT_2,MSxc_5,MDxc_2	-2.88	33.06	22.97		1	22.97**
MD	-.63	1.56	1.09 △		-	
MS	1.38	7.56	5.25		1	5.25*
Error	--	2.68	1.87	5.14	6	0.857
Total	--	143.94	100.0	--	15	---

Table 26. Wave Propagation

The first car maneuver is:	Start	Peak of Speed	Stop
	T = 1.25, L = 0	T = 3.0, S = 12.0, L = 10.5	T = 5.0, L = 24.0

Run	Car #	Start			Peak of Speed				Stop		
		Time	Location	Propag. Speed	Time	Speed	Location	Propag. Speed	Time	Location	Propag. Speed
1		5.25	-80	20.0	6.5	4.7	-78.6	25.5	7.5	-76.0	40.0
	9	10.25	-180	20.0	10.75	0.7	-179.9	24.6	11.5	-179.4	31.3
2	4	5.25	-80	20.0	6.0	4.9	-78.7	29.7	6.5	-77.3	67.5
	9	9.75	-180	21.2	10.0	0.7	-179.9	27.2	10.25	-179.5	38.8
3	4	5.25	-80	20.0	6.5	4.6	-78.5	25.4	7.0	-76.7	50.4
	9	10.25	-180	20.0	10.75	0.5	-179.9	24.6	11.75	-179.6	30.2
4	4	5.25	-80	20.0	6.0	7.4	-78.6	29.7	6.75	-74.7	56.4
	9	9.5	-180	21.8	10.0	5.3	-177.8	26.9	10.25	-176.4	38.2
5	4	7.75	-80	12.3	8.25	3.6	-78.3	16.9	8.75	-77.1	27.0
	9	13.50	-180	14.7	13.75	1.2	-179.8	17.7	14.50	-179.4	21.4
6	4	7.5	-80	12.8	8.75	8.1	-74.3	14.7	9.5	-71.0	21.1
	9	13.75	-180	14.4	14.5	3.2	-176.7	16.3	15.0	-175.9	20.0
7	4	7.0	-80	13.9	8.25	3.9	-78.3	16.9	8.75	-76.7	26.9
	9	13.50	-180	14.7	15.0	1.2	-178.9	15.8	15.5	-178.5	19.3
8	4	7.75	-80	12.3	8.0	3.0	-79.5	18.0	8.5	-78.5	22.4
	9	13.25	-180	15.0	14.25	2.4	-178.3	16.8	14.75	-177.5	20.7
9	4	5.25	-80	20.0	8.25	21.9	-53.8	12.2	(9.25)	-36.8	(14.3)
	9	10.25	-180	20.0	14.75	26.9	-135.2	12.4	(16.0)	-107.0	(11.9)
10	4	5.25	-80	20.0	8.0	27.7	-44.1	10.9	(8.5)	-31.2	(15.8)
	9	10.25	-180	20.0	14.25	37.3	-111.1	10.8	(14.75)	-93.3	(12.0)
11	4	5.25	-80	20.0	8.5	16.8	-57.0	12.3	9.75	-45.8	14.7
	9	10.25	-180	20	15.0	21.1	-143.3	128	16.25	-124.1	13.2
12	4	5.25	-80	20.0	8.0	18.1	-57.1	13.5	(9.5)	-39.9	(14.2)
	9	10.25	-180	20.0	14.25	23.7	-137.1	13.1	(15.75)	-110.0	(12.5)

Table 26. Wave Propagation

The first car maneuver is: Start T = 1.25, L = 0   Peak of Speed T = 3.0, S = 12.0, L = 10.5   Stop T = 5.0, L = 24.0

		Start			Peak of Speed				Stop		
Run	Car #	Time	Location	Propag. Speed	Time	Speed	Location	Propag. Speed	Time	Location	Propag. Speed
1		5.25	-80	20.0	6.5	4.7	-78.6	25.5	7.5	-76.0	40.0
	9	10.25	-180	20.0	10.75	0.7	-179.9	24.6	11.5	-179.4	31.3
2	4	5.25	-80	20.0	6.0	4.9	-78.7	29.7	6.5	-77.3	67.5
	9	9.75	-180	21.2	10.0	0.7	-179.9	27.2	10.25	-179.5	38.8
3	4	5.25	-80	20.0	6.5	4.6	-78.5	25.4	7.0	-76.7	50.4
	9	10.25	-180	20.0	10.75	0.5	-179.9	24.6	11.75	-179.6	30.2
4	4	5.25	-80	20.0	6.0	7.4	-78.6	29.7	6.75	-74.7	56.4
	9	9.5	-180	21.8	10.0	5.3	-177.8	26.9	10.25	-176.4	38.2
5	4	7.75	-80	12.3	8.25	3.6	-78.3	16.9	8.75	-77.1	27.0
	9	13.50	-180	14.7	13.75	1.2	-179.8	17.7	14.50	-179.4	21.4
6	4	7.5	-80	12.8	8.75	8.1	-74.3	14.7	9.5	-71.0	21.1
	9	13.75	-180	14.4	14.5	3.2	-176.7	16.3	15.0	-175.9	20.0
7	4	7.0	-80	13.9	8.25	3.9	-78.3	16.9	8.75	-76.7	26.9
	9	13.50	-180	14.7	15.0	1.2	-178.9	15.8	15.5	-178.5	19.3
8	4	7.75	-80	12.3	8.0	3.0	-79.5	18.0	8.5	-78.5	22.4
	9	13.25	-180	15.0	14.25	2.4	-178.3	16.8	14.75	-177.5	20.7
9	4	5.25	-80	20.0	8.25	21.9	-53.8	12.2	(9.25)	-36.8	(14.3)
	9	10.25	-180	20.0	14.75	26.9	-135.2	12.4	(16.0)	-107.0	(11.9)
10	4	5.25	-80	20.0	8.0	27.7	-44.1	10.9	(8.5)	-31.2	(15.8)
	9	10.25	-180	20.0	14.25	37.3	-111.1	10.8	(14.75)	-93.3	(12.0)
11	4	5.25	-80	20.0	8.5	16.8	-57.0	12.3	9.75	-45.8	14.7
	9	10.25	-180	20	15.0	21.1	-143.3	128	16.25	-124.1	13.2
12	4	5.25	-80	20.0	8.0	18.1	-57.1	13.5	(9.5)	-39.9	(14.2)
	9	10.25	-180	20.0	14.25	23.7	-137.1	13.1	(15.75)	-110.0	(12.5)

Table 26. (Continued)

Run	Car #	Start			Peak of Speed				Stop		
		Time	Location	Propag. Speed	Time	Speed	Location	Propag. Speed	Time	Location	Propag. Speed
13	4	9.25	-80	10.0	10.25	7.9	-75.5	11.9	11.5	-70.4	14.5
	9	16.75	-180	11.6	17.75	6.8	-175.6	12.6	18.5	-172.4	14.5
14	4	9.0	-80	10.3	10.5	9.1	-71.9	11.0	11.5	-66.4	13.9
	9	16.5	-180	11.8	17.5	7.7	-175.5	12.8	18.0	-172.6	15.1
15	4	9.0	-80	10.3	10.25	14.9	-69.0	11.0	12.25	-50.8	10.3
	9	18.0	-180	10.7	19.25	14.6	-169.0	11.0	20.5	-157.1	11.7
16	4	8.75	-80	10.7	9.75	14.8	-72.5	12.3	10.75	-61.0	14.8
	9	16.5	-180	11.8	17.5	13.9	-172.5	12.6	18.25	-165.3	14.3

Note: ( ) shows that collision occurred

ANOVA for Wave Propagation: Stop-Run-Stop Case

Table 27. Wave Speed of Start Point df of Error = 12

	Observation of #4 Car				Observation of #9 Car			
Source	Effect	SS	%of SS	MS/Total SS	Effect	SS	%ofSS	MS/Total SS
T_1	-8.43	283.92	95.15	95.15**	-7.29	212.43	89.33	89.33**
T_2	-1.25	6.25	2.09	2.09**	-1.99	15.80	6.64	6.64**
T_1xT_2	-1.25	6.25	2.09	2.09**	-1.23	6.13	2.58	2.58**
Error	---	1.96	0.67	0.056	---	3.43	1.44	0.120

($\overline{X}_4$= 16.84 ft/s) ($\overline{X}_9$ = 16.73 ft/s)

Table 28. Wave Speed of Peak Point df of Error = 12

	Observation of #4 Car				Observation of #9 Car			
Source	Effect	SS	%ofSS	MS/Total SS	Effect	SS	%ofSS	MS/Total SS
T_1	-5.81	135.14	19.69	19.69**	-4.60	84.64	16.80	16.80**
T_2	-10.21	417.18	60.78	60.78**	-8.96	322.20	63.95	63.95**
T_1xT_2	5.14	105.58	15.38	15.38**	4.58	83.72	16.62	16.62**
Error	---	28.49	4.15	0.346	--	13.28	2.63	0.219

($\overline{X}_4$ = 16.99 ft/s) ($\overline{X}_9$ = 16.75 ft/s)

ANOVA for Wave Propagation: Stop-Run-Stop Case - Continued

Table 29. Wave Speed of Stop Point df of Error = 10

	Observation of #4 Car				Observation of #9 Car			
Source	Effect	SS	%ofSS	MS/TotalSS	Effect	SS	%ofSS	MS/TotalSS
c_2	3.50	49.0	1.06	1.06	2.36	22.33	1.69	1.69*
T_1	-15.30	936.36	20.22	20.22**	-6.29	158.13	11.98	11.98**
T_2	-24.90	2480.04	53.56	53.56**	-14.44	833.77	63.19	63.19**
T_1xT_2	13.93	775.62	16.75	16.75**	7.99	255.20	19.34	19.34**
MD	5.85	136.89	2.96	2.96*	2.31	21.39	1.62	1.62*
Error	---	252.89	5.46	0.546	---	28.71	2.18	0.218

($\overline{X}_4$ = 26.5 ft/s) ($\overline{X}_9$ = 20.27 ft/s)

F_1, 12(0.05) = 4.75 F_1,12(0.01) = 9.33

F_1, 10(0.05) = 4.96 F_1,10(0.01) = 10.0

Table 30. Amplitude of Speed at the Peak Point

Run	Car #	Speed	Amplitude Ratio	Amp. Ratio Vehicle
1	4 9	4.7 0.7	.39 0.58	.79 .73
2	4 9	4.9 0.7	.41 .058	.80 .75
3	4 9	4.6 0.5	.38 .42	.79 .70
4	4 9	7.4 5.3	.62 .44	.89 .91
5	4 9	3.6 1.2	.30 .10	.74 .77
6	4 9	8.1 3.2	.68 .27	.90 .86
7	4 9	3.9 1.2	.33 .10	.75 .77
8	4 9	3.0 2.4	.25 .20	.71 .84
9	4 9	21.9 26.9	1.83 2.24	1.16 1.09
10	4 9	27.7 37.3	2.31 3.11	1.23 1.13
11	4 9	16.8 21.1	1.40 1.76	1.09 1.07
12	4 9	18.1 23.7	1.51 1.98	1.11 1.08
13	4 9	7.9 6.8	.66 .57	.90 .94
14	4 9	9.1 7.7	.76 .64	.93 .95
15	4 9	14.9 14.6	1.24 1.22	1.05 1.02

Table 30. (Continued)

Run	Car #	Speed	Amplitude Ratio	Amp. Ratio Vehicle
16	4	14.8	1.23	1.05
	9	13.9	1.16	1.01

$$\text{Amplitude Ratio} = \frac{\#4 \text{ or } \#9 \text{ Peak Speed}}{\#0 \text{ car Peak Speed}}$$

$$\text{Amplitude Ratio/Vehicle} = (\text{Amplitude Ratio})^{1/n}$$

where $n = 4$ or 9

Table 31. ANOVA for Amplitude: Stop-Run-Stop Case

	Observation of $4 Car				Observation of #9 Car			
Source	Effect	SS	%ofSS	MS	Effect	SS	%ofSS	MS
c_2	5.50	121.0	3.92	3.92*	4.38	76.56	1.89	1.89
T_1	-3.75	56.25	1.82	1.82	-10.38	430.56	10.60	10.60**
T_2	24.50	2401.0	77.86	77.86**	26.88	2889.06	71.14	71.14**
T_1xT_2	-7.5	225.0	7.30	7.30**	-6.13	150.06	3.69	3.69*
MXS	4.5	81.0	2.63	2.63	9.13	333.06	8.20	8.20**
Error	---	199.5	6.47	.647	---	181.64	4.47	.447

($\overline{X}_4$ = 91.38%) ($\overline{X}_9$ = 93.06%)

inant factor, and a longer T_1, combined with a shorter T_2; in other words, more patience in acceleration and quicker response when decelerating, result in a safer and more stable situation.

Merging (22, 23)

As the third application analysis of the merging situation on a highway ramp was selected. In the former two applications, the interact was concentrated mainly on analyzing the relationship between driver characteristics or car performance limits and safety or stability in specific maneuvers. In the merging situation, there are some other important subjects to be analyzed, such as merging lane length and success rate of merging, or the effectiveness of the device which tells the driver on the ramp of the existence of an acceptable gap, etc. It may be possible to deal with these subjects by expanding this simulation model; however, at this time the interest is focused on behavior of vehicles on the righthand side of the main flow from the time just after a single vehicle has merged under various conditions. A 2^4 factorial experimental design was applied to this analysis.

Design of Experiment

Situation. The following situation is assumed here:

(a) The simulation will start just after vehicle #1 has merged between car #0 and car #2,

(b) The merging vehicle (#1) will use its maximum acceleration when it is merging.

(c) The characteristics of drivers and cars,,except for the maximum acceleration of the #1 vehicle, are fixed at their typical values as previously defined.

Factors. As shown in Table 32, four factors, i.e., the main flow speed; the acceptance gap, which is the headway time between car #0 and car #2; the acceleration capacity; and the relative speed between the merging vehicle and the main flow speed. According to Figure 2 in (22), the rate of acceptance is 7, 45, and 80 percent when the gap is 2.0, 3.0, and 4.0 seconds respectively. Consequently, 2.0 and 4.0 seconds were chosen as the two levels of gap. The typical truck and car acceleration capacities were chosen as levels of acceleration. The combination of factors used in the design of the experiment is shown in Table 32.

Results of Simulation

Results of the simulation are summarized in Tables 33 and 34. Two kinds of measurements were applied to analysis; one is total collision speed as before and the other is total disturbance distance, which is calculated by subtracting actual travel distance from ordinary travel distance without the disturbance of the merging vehicle, #1. This is also a measure of the degree of influence of the merging vehicle upon the main flow.

Results of ANOVA are shown in Tables 35 - 37. In all cases, the gap is the most significant factor. It is very interesting here that the factor of maximum acceleration is not significant.

Flow in a Bottleneck

The last application merely shows the possibilities for analysis of the relationship between traffic bottlenecks and traffic flow.

Many bottlenecks can be seen in traffic; they include tunnels,

Table 32. Combination of Factors for Merging Experiment

					Auxiliary Parameters which are determined by Factors			
Factor / Run	MFS	GAP	ACC	RS	SMC	GAPL	IDST	IL1
1	50	2	T	20	30	100	80	50
2	80	2	T	20	60	160	116	80
3	50	4	T	20	30	200	80	50
4	80	4	T	20	60	320	116	80
5	50	2	c	20	30	100	80	50
6	80	2	c	20	60	160	116	80
7	50	4	c	20	30	200	80	50
8	80	4	c	20	60	320	116	80
9	50	2	T	40	10	100	80	50
10	80	2	T	40	40	160	116	80
11	50	4	T	40	10	200	80	50
12	80	4	T	40	40	320	116	80
13	50	2	c	40	10	100	80	50
14	80	2	c	40	40	160	116	80
15	50	4	c	40	10	200	80	50
16	80	4	c	40	40	320	116	80

Note: MFS is the main flow speed [ft/s]
GAP is the headway between car #0 and car #2 [s]
ACC is the maximum acceleration of a car or a truck. [ft/s²]

as car: $\dot{v} = 0.07(140-v)$
truck: $\dot{v} = 0.04(100-v)$

RS is the relative speed [ft/s]
SMC is the speed of the merging car (#1) [ft/s]
GAPL is the gap length [ft]
IDST is the initial distance between following cars (#2 ~ #9) [ft]
IL1 is the initial location of the merging car (#1) [ft]

Table 33. Summary of Collision Time and Speed in Merging

Run	Crash	Car # 1	Car # 2	Car # 3	Car # 4	Car # 5	Car # 6	Car # 7	Car # 8	Car # 9	No. of Crashes Total
1	Time										0
	Speed										0
2	Time					9.5	10.5	11.5	12.75	14.0	5
	Speed					27.0	38.0	48.1	52.0	55.0	220.1
3	Time										0
	Speed										0
4	Time										0
	Speed										0
5	Time										0
	Speed										0
6	Time							13.0	14.0	15.0	3
	Speed							28.8	37.9	46.7	113.4
7	Time										0
	Speed										0
8	Time										0
	Speed										0
9	Time		1.0	2.5	4.25	5.75	7.0	8.5	10.0	11.75	8
	Speed		50.1	35.0	25.1	21.9	21.7	16.8	12.6	29.5	212.7
10	Time		1.75	3.25	4.75	6.25	7.5	8.75	10.0	11.25	8
	Speed		75.1	64.5	61.6	59.6	61.8	62.0	64.1	67.3	516.0

Table 33. (Continued)

Run	Crash	Car # 1	Car # 2	Car # 3	Car # 4	Car # 5	Car # 6	Car # 7	Car # 8	Car # 9	No. of Crashes Total
11	Time Speed										0 0
12	Time Speed						13.5 31.2	14.5 43.2	15.75 49.0	17.0 52.0	4 175.4
13	Time Speed		1.0 50.1	2.5 35.0	4.25 25.1	5.75 21.9	7.0 21.7	8.5 16.8	10.0 12.6	11.75 29.5	8 212.7
14	Time Speed		2.25 65.1	3.75 61.7	5.25 60.4	6.5 64.2	7.75 63.9	9.0 63.0	10.25 64.3	11.50 67.3	8 509.9
15	Time Speed										0 0
16	Time Speed										0 0

Table 34. Disturbance in Travel Distance By Merging Vehicle

Run	Final Location Traveled Distance	#2	# 3	# 4	# 5	# 6	# 7	# 8	# 9	TDD*
1	FL [ft]	1039	954	864	732	580	434	303	184	
	TD [ft]	1139	1134	1124	1072	1000	934	883	844	183.75
2	FL	1573	1442	172	160	153	140	136	131	
	TD	1733	1718	562	668	777	880	992	1103	865.88
3	FL	1040	956	877	792	694	551	392	248	
	TD	1240	1236	1237	1232	1214	1151	1072	1008	26.25
4	FL	1574	1445	1311	1173	1039	930	854	769	
	TD	1894	1881	1863	1841	1823	1830	1840	1901	57.13
5	FL	1047	961	872	737	583	438	306	187	
	TD	1147	1141	1132	1077	1003	938	886	847	178.63
6	FL	1697	1587	1364	1048	225	212	204	193	
	TD	1857	1863	1756	1556	849	952	1060	1165	577.75
7	FL	1048	963	878	791	693	612	526	416	
	TD	1248	1243	1238	1231	1213	1212	1206	1176	-20.88
8	FL	1698	1579	1455	1342	1260	1149	991	819	
	TD	2018	2015	2007	2010	2044	2049	2007	1951	-92.63
9	FL	-50	062	-72	-83	-98	-111	-125	-140	
	TD	50	118	188	257	322	389	455	520	912.63
10	FL	-21	-24	-29	-29	-38	-47	-57	-70	
	TD	139	252	363	479	586	693	799	902	1393.38

Table 34. (Continued)

Run	Final Location Traveled Distance	# 2	# 3	# 4	# 5	# 6	# 7	# 8	# 9	TDD*
11	FL	845	732	623	527	431	310	185	69	
	TD	1045	1012	983	967	951	910	865	829	254.75
12	FL	1270	1148	992	224	212	200	193	186	
	TD	1590	1584	1544	92	996	1100	1209	1318	640.88
13	FL	-50	-62	-72	-83	-98	-111	-125	-140	
	TD	50	118	188	257	322	389	455	520	912.63
14	FL	128	128	92	-54	-166	-276	-370	-495	
	TD	288	404	484	454	458	464	486	477	1480.63
15	FL	1042	949	803	627	468	323	194	78	
	TD	1242	1229	1163	1067	988	923	874	838	159.5
16	FL	1706	1600	1433	1231	1048	888	759	661	
	TD	2026	2036	1985	1899	1832	1788	1775	1793	28.5

* Total Disturbed Distance

Table 35. ANOVA for Collision Speed in Merging

($\overline{X}$ = 122.5 ft/sec)

Source	Effect	df	SS	% of SS	MS/Total SS
MFS	138.68	1	76923.0	16.41	16.41**
GAP	-201.18	1	161885.5	34.54	34.54**
MFSxGAP	-94.83	1	35967.1	7.67	7.67**
ACC	-36.03	1	5191.2	1.11	1.11
MFSxACC	-36.03	1	519.12	1.11	1.11
GAPxACC	-7.83	1	244.9	.05	.05
RS	161.63	1	104522.9	22.30	22.30**
MFSxRS	55.30	1	12232.4	2.61	2.61
GAPxRS	-117.80	1	55507.4	11.84	11.84*
ACCxRS	-9.35	1	349.7	.07	.07
Error	---	5	10641.0	2.27	.454
Total	---	15	468656.3	100.0	---

Table 36. ANOVA for Number of Collisions in Merging

($\bar{X}$ = 2.75 vehicle)

Source	Effect	df	SS	% of SS	MS/Total SS
MFS	1.5	1	9.0	4.86	4.86
GAP	-4.5	1	81.0	43.78	43.78**
MFSxGAP	-.5	1	1.0	.54	.54
ACC	-.75	1	2.25	1.22	1.22
MFSxACC	-.75	1	2.25	1.22	1.22
GAPxACC	-.25	1	.25	.14	.14
RS	3.5	1	49.0	26.49	26.49**
MFSxRS	-.5	1	1.0	.54	.54
GAPxRS	-2.5	1	25.0	13.51	13.51*
ACCxRS	-.25	1	.25	.14	.14
Error	---	5	14.0	7.568	1.514
Total	---	15	185.0	100.0	---

Table 37. ANOVA for Disturbance

($\overline{X}$ = 4724 ft)

Source	Effect	df	SS	% of SS	MS/ Total SS
MFS	293.04	1	343,484	8.82	8.82*
GAP	-681.46	1	1,857,565	47.72	47.72**
MFSxGAP	-239.49	1	229,417	5.89	5.89*
ACC	-138.74	1	76,992	1.98	1.98
MFSxACC	-101.81	1	41,463	1.07	1.07
GAPxACC	-87.41	1	30,564	.79	.79
RS	500.86	1	1,003,453	25.78	25.78**
MFSxRS	32.94	1	4,339	.11	.11
GAPxRS	-222.41	1	197,869	5.08	5.08*
ACCxRS	-16.19	1	1,048	.03	.03
Error	---	5	106,440	2.734	.547
Total	---	15	3,892,634	100.0	---

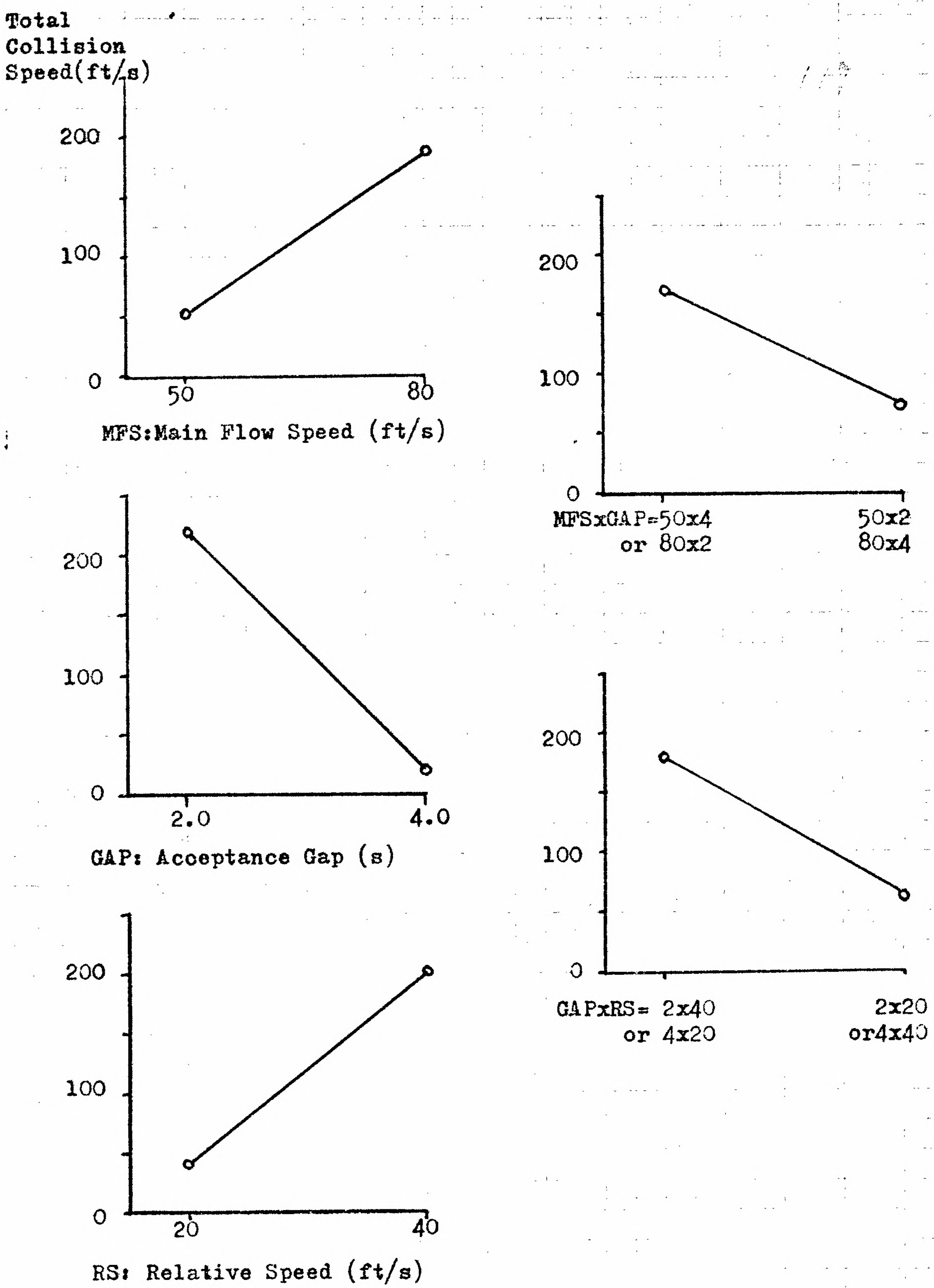


Figure 22. Effect of Factors on Collision Speed in Merging

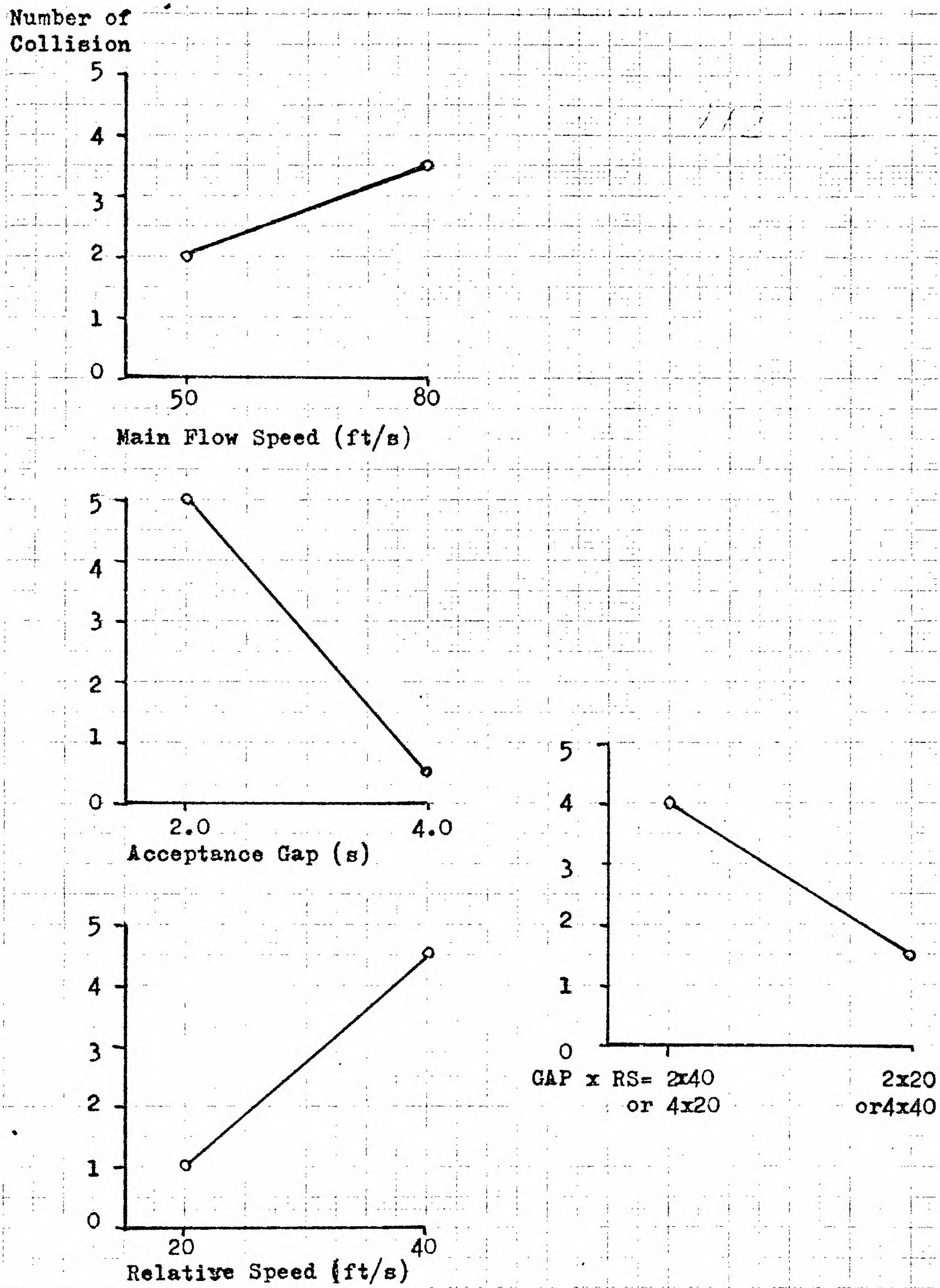


Figure 23. Effect of Factors on Number of Collisions in Merging

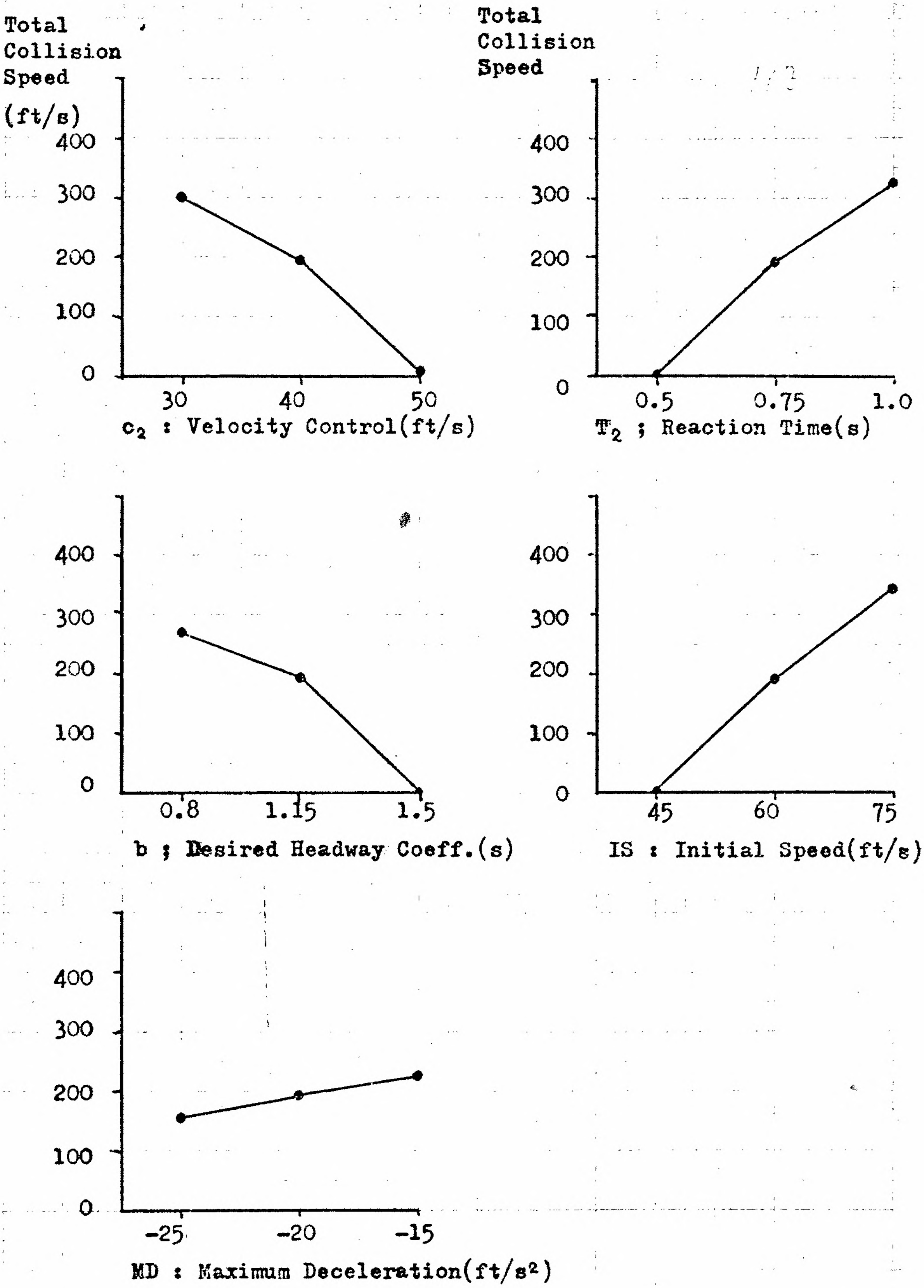


Figure 20. Total Collision Speed in Emergency Stop

bridges, an area under construction, traffic accidents, etc. In attempting to deal with the bottleneck problem, so many factors, such as the main flow speed, the bottleneck speed, the bottleneck shape, the capacity of cars, and drivers' characteristics, must be considered, that the bottleneck problem cannot be generalized easily and effectively. It might be beyond the scope of this research to analyze an actual bottleneck problem; therefore, only the bottleneck shape under specific conditions will be discussed.

Problem

Let us assume the problem of a bottleneck where the vehicles h have to reduce their speed from 80 feet per second to 40 feet per second. In this situation, how does the bottleneck shape, more exactly pre-bottleneck shape, affect the flow of vehicles? It is usually possible to select shapes of pre-bottlenecks, in other words, control speeds of vehicles before the bottleneck by means of placing speed limit signs or other devices.

Here three bottleneck shapes are selected, as shown in Figure 25.

Evaluation

The results of the runs are shown in Appendix C. The flow efficiency may be measured in terms of how far the vehicles can travel for a certain time period. Let us focus on how far car #9 has run for 50 seconds because it is obvious that for the first car (#0), the steeper slope of the pre-bottleneck shape is the better in the sense of traveled distance. In 50 seconds, car #9 ran 3,249 feet in the case of shape 1, 3,364 feet in the case of shape 2, 3,454 feet in the case of shape

3. Thus, shape 3 looks best; however, in the case of shape 3, the minimum speed of car #8 reached 32.8 ft/s and that of car #9 was 25.7 ft/s, while the speed reduction in the cases of shapes 2 and 3 is not more than the limit of the bottleneck itself, i.e., 40 ft/s. Then, if the platoon is long enough, the cars which belong to the latter part of the platoon will be forced to stop, and a congested situation will be created. Therefore, shape 3 may not be desirable in the case where longer platoons are expected to pass this bottleneck.

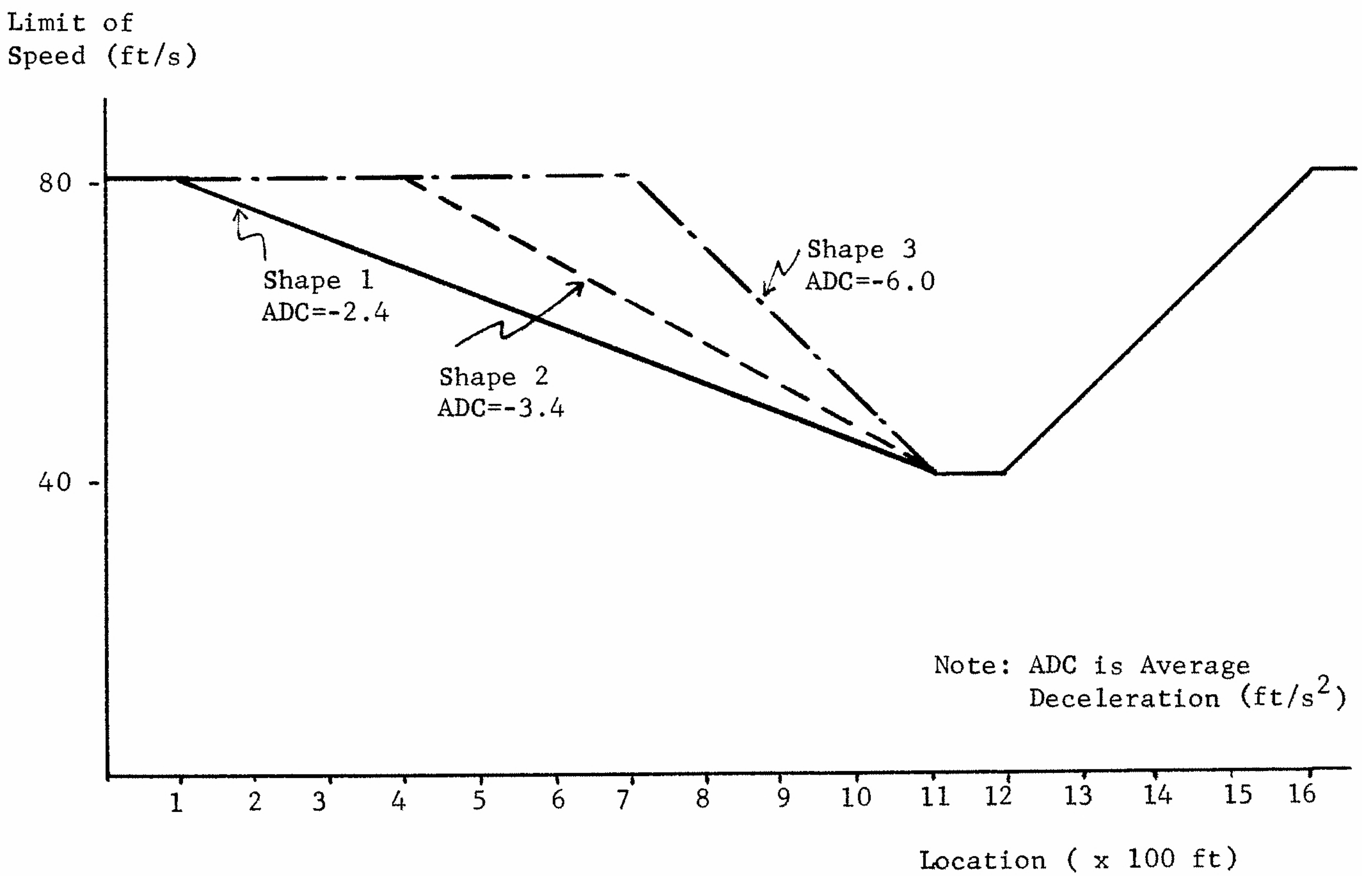


Figure 25. Bottleneck Shapes

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Research Results

In this thesis, car following behavior has been researched from various aspects. Based on previous research, experiments and data collection, there has been developed a car following simulation model, which possesses wider adaptability and greater reality than the published mathematical models.

Characteristics of the Model

The characteristics of the developed simulation model can be described as follows:

Position in Traffic Models. According to (24), traffic models are classified and organized as in Figure 26. By this classification, this simulation model fits into the upper left part of the figure, although it is digital simulation. In comparison with other kinds of traffic models, it can be said that this model focuses on the detailed behavior of each vehicle.

Verification. The basis of this model, i.e., the Greenberg model, has been verified by experiments, and almost all component parts consist of results of observation and experimental data. Concerning the appropriateness of the overall model structure, virtually all relationships are based on a physical system that can be verified. Further, base simulation runs of the model agree well with published ob-

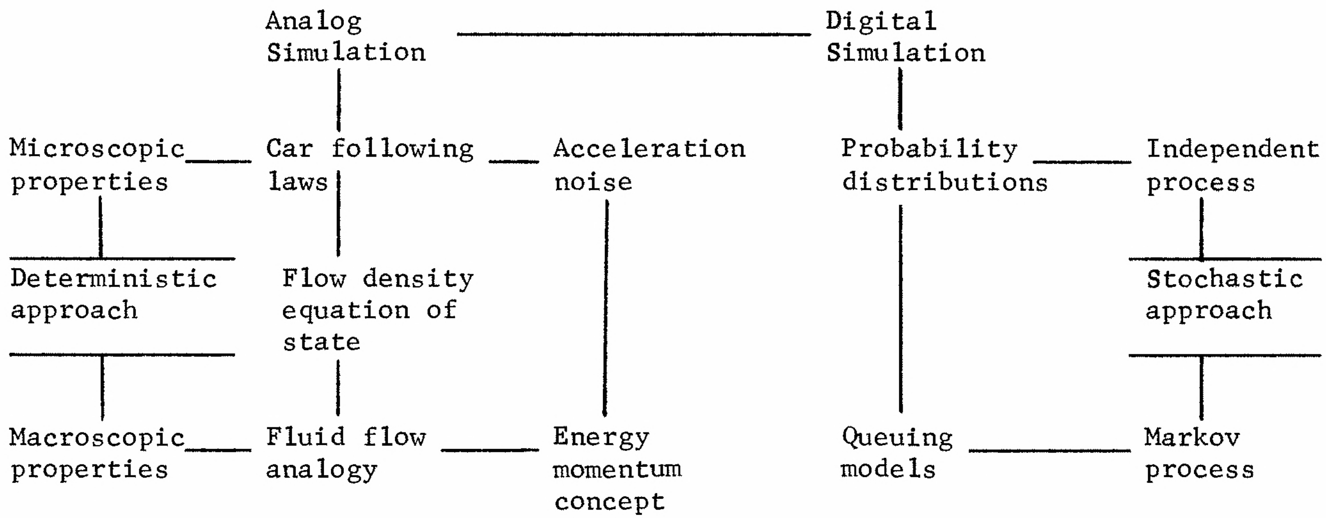


Figure 26. Classification and Organization of Traffic Models from (24)

served data.

Limits and Adaptability. The first main limit or restriction of this model is that it deals with cars on a single lane, so no passing is allowed. Therefore, such behavior as lane changing and passing cannot be handled. However, we can deal with the car following situation just prior to and just after passing or lane changing.

The second restriction is the computer capacity and simulation time. As shown in the appendix, this simulation program consists of about 500 statements in DYNAMO. The number of statements is almost 50 times the number of cars to be dealt with. Of course, it depends on the capacity of the computer, but it may be very difficult for this model to deal with more than 100 vehicles. The simulation computation time depends on the capacity of the computer, the period required to execute simulation (more exactly, the number of repeat calculations, which is obtained by dividing the period by DT), and the scale of the program (in this case the number of cars to be dealt with). In the case of this program, which deals with ten cars and uses the UNIVAC 1108 at Georgia Tech, it requires about 65 seconds to compile and 25 seconds to simulate a 24-second run. Although this simulation model may not be efficient enough to deal with many vehicles statistically, that is not its purpose.

The third point is the assumption of the driver's response. In this model, the driver is assumed to respond only to the behavior of the car in front of him. But it is technically easy to change the structure so that the driver responds to the second car ahead, the third car ahead, etc. Consequently, this is not a restriction. This program has considerable flexibility beyond the first and the second restriction because it is written in DYNAMO. The driver's response structure and all parameters can easily be changed. As shown in Chapter IV, this model has the potential for application to the analysis of various aspects of car following.

Result of Applications

As mentioned in detail in Chapter IV, the reaction time appears as a very important factor for stability and safety in all applications. The results of analysis show that faster response to deceleration and a response to acceleration increases the safety. In both the emergency stop case and the merging case, the initial speed and main flow speed are significant to the collision speed.

It is interesting that the performance of the car, i.e., acceleration and deceleration ability on a normal dry road does not tend to be as significant as reaction time and initial speed.

Recommendations for Further Research

There are two possible directions for further research. One is more detailed research on the structure or components of this simulation model. In particular, research into and application of another type of car following equation would be very interesting and important; for example:

$$\ddot{X}_n(t+T) = \lambda_1(\dot{X}_{n-1}(t) - \dot{X}_n(t)) + \lambda_2(\dot{X}_{n-2}(t) - \dot{X}_n(t)) + \cdots \quad (5\text{-}1)$$

The other suggested direction for further research is more detailed application of this model. The main purpose of this research was the development of a realistic car following model; thus, this research was intended for general study of wider applications rather than narrower and deeper study of specific problems. However, many interesting subjects might be found for application of this model; some of them are:

(i) The effect of a violent driver in the platoon in a specific situation

(ii) The effect of the mixture of passenger cars and trucks.

(iii) The influence of such disturbances as mist, snow, rain, or malfunction of brake lights.

(iv) The influence of the geometric configuration of the road.

(v) The effect of improvements in brake light design.

(vi) Analysis of an acceptance gap notifying device.

APPENDIX A

NOTATION

Symbol	Description	Unit
$X_n(t)$	position of nth vehicle at time t	ft
$\dot{X}_n(t)$	velocity of nth vehicle at time t	ft/s
$\ddot{X}_n(t)$	acceleration of nth vehicle at time t	ft/s^2
T	reaction time	s
T_1	reaction time when front car's brake light is off	s
T_2	reaction time when front car's brake light is on	s
λ	sensitivity	1/s
c_1	sensitivity coefficient of Chandler et al. model	1/s
c_2	sensitivity coefficient of Greenberg model and also velocity control in simulation model	ft/s
c_3	sensitivity coefficient of Greenshields model	ft/s^2
c_4	sensitivity coefficient of Edie model	ft
c_5	headway control in simulation model	$1/s^2$
q	flow rate = uk	vhc/s
qm	maximum flow rate	vhc/s
k	concentration or density	vhc/ft
k_j	jam concentration	vhc/ft
u	velocity (mainly used in steady state)	ft/s
uf	free velocity	ft/s
uop	optimum velocity	f/s
r	acceleration coefficient	--

Symbol	Description	Unit
D	desired headway = $a + b\dot{X}$	ft
a	minimum headway	ft
b	desired headway coefficient	s
MA	maximum acceleration = $\alpha(MS - \dot{X})$	ft/s^2
MS	maximum speed	ft/s
α	acceleration coefficient	1/s
MD	maximum deceleration	ft/s^2
IS	initial speed	ft/s
ID	initial distance (headway)	ft
σ_D	acceleration noise	ft/s^2

APPENDIX B

A SAMPLE OF PROGRAM

```
@@@@@   @   @     @@@   @@@@@   @   @   @   @   @@@@     @@@
@       @   @      @      @     @@ @@   @   @   @   @   @   @
@       @   @      @      @     @ @ @   @   @   @   @   @   @
@@@     @   @      @      @     @   @   @   @   @@@@    @@@@@   @@@@@
@       @   @      @      @     @   @   @   @   @ @     @   @
@       @   @   @  @      @     @   @   @   @   @  @    @   @
@        @@@     @@     @@@@@   @   @    @@@    @   @   @   @

12345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012
@RUN IEADV,031A0935,FUJIMURA-M,10,200
@ASG,T  20
@ASG,T  24,///500
@ASG,T  14
@ASG,A GT*DYNAMO
FAC WARNING        040200004000
@XQT GT*DYNAMO.DYNAMO


 DYNAMO LEVEL B GEORGIA TECH REV. 1.2A


 RUN    FUJI-1
 NOTE
 NOTE      M=0,L=1 MODEL
 NOTE      CONSIDERING DESIRED DISTANCE
 NOTE      AND THE EFFECT OF BRAKE LIGHTS
 NOTE
 NOTE      LEVEL
 NOTE
 1L     S0.K=S0.J+(DT)(AA0.JK+0)             SPEED OF CAR#0
 1L     S1.K=S1.J+(DT)(AA1.JK+0)             SPEED OF CA R
 1L     S2.K=S2.J+(DT)(AA2.JK+0)             SPEED OF CA R
 1L     S3.K=S3.J+(DT)(AA3.JK+0)             SPEED OF CA R
 1L     S4.K=S4.J+(DT)(AA4.JK+0)             SPEED OF CA R
 1L     S5.K=S5.J+(DT)(AA5.JK+0)             SPEED OF CA R
 1L     S6.K=S6.J+(DT)(AA6.JK+0)             SPEED OF CA R
 1L     S7.K=S7.J+(DT)(AA7.JK+0)             SPEED OF CAR
```

```
1L     S8.K=S8.J+(DT)(AA8.JK+0)          SPEED OF CAR
1L     S9.K=S9.J+(DT)(AA9.JK+0)          SPEED OF CAR
NOTE
1L     L0.K=L0.J+(DT)(SS0.JK+0)          POSITION OF CAR#0
1L     L1.K=L1.J+(DT)(SS1.JK+0)          POSITION OF CAR
1L     L2.K=L2.J+(DT)(SS2.JK+0)          POSITION OF CAR
1L     L3.K=L3.J+(DT)(SS3.JK+0)          POSITION OF CAR
1L     L4.K=L4.J+(DT)(SS4.JK+0)          POSITION OF CAR
1L     L5.K=L5.J+(DT)(SS5.JK+0)          POSITION OF CAR
1L     L6.K=L6.J+(DT)(SS6.JK+0)          POSITION OF CAR
1L     L7.K=L7.J+(DT)(SS7.JK+0)          POSITION OF CAR
1L     L8.K=L8.J+(DT)(SS8.JK+0)          POSITION OF CAR
1L     L9.K=L9.J+(DT)(SS9.JK+0)          POSITION OF CAR
NOTE
NOTE      AUXILIARY EQ.
NOTE
6R     R1R1.KL=RPS1.K                    DELAY .25 SEC
6R     R2R1.KL=RPS2.K                    DELAY .25 SEC
6R     R3R1.KL=RPS3.K                    DELAY .25 SEC
6R     R4R1.KL=RPS4.K                    DELAY .25 SEC
6R     R1R2.KL=R1R1.JK                   DELAY .50 SEC
6R     R2R2.KL=R2R1.JK                   DELAY .50 SEC
6R     R3R2.KL=R3R1.JK                   DELAY .50 SEC
6R     R4R2.KL=R4R1.JK                   DELAY .50 SEC
6R     R4R3.KL=R4R2.JK                   DELAY .75 SEC
6R     R3R3.KL=R3R2.JK                   DELAY .75 SEC
6R     R2R3.KL=R2R2.JK                   DELAY .75 SEC
6R     R1R3.KL=R1R2.JK                   DELAY .75 SEC
6R     R1R4.KL=R1R3.JK                   DELAY 1.00 SEC
6R     R2R4.KL=R2R3.JK                   DELAY 1.00 SEC
6R     R3R4.KL=R3R3.JK                   DELAY 1.00 SEC
6R     R4R4.KL=R4R3.JK                   DELAY 1.00 SEC
6R     R4R5.KL=R4R4.JK                   DELAY 1.25 SEC
6R     R3R5.KL=R3R4.JK                   DELAY 1.25 SEC
6R     R2R5.KL=R2R4.JK                   DELAY 1.25 SEC
6R     R1R5.KL=R1R4.JK                   DELAY 1.25 SEC
6R     R1R6.KL=R1R5.JK                   DELAY 1.50 SEC
6R     R2R6.KL=R2R5.JK                   DELAY 1.50 SEC
6R     R3R6.KL=R3R5.JK                   DELAY 1.50 SEC
6R     R4R6.KL=R4R5.JK                   DELAY 1.50 SEC
6R     R4R7.KL=R4R6.JK                   DELAY 1.75 SEC
6R     R3R7.KL=R3R6.JK                   DELAY 1.75 SEC
6R     R2R7.KL=R2R6.JK                   DELAY 1.75 SEC
6R     R1R7.KL=R1R6.JK                   DELAY 1.75 SEC
6R     R5R1.KL=RPS5.K                    DELAY .25 SEC
6R     R6R1.KL=RPS6.K
6R     R7R1.KL=RPS7.K
6R     R8R1.KL=RPS8.K
6R     R9R1.KL=RPS9.K
6R     R5R2.KL=R5R1.JK
6R     R6R2.KL=R6R1.JK
6R     R7R2.KL=R7R1.JK
6R     R8R2.KL=R8R1.JK
6R     R9R2.KL=R9R1.JK
6R     R5R3.KL=R5R2.JK                   DELAY .75 SEC
6R     R6R3.KL=R6R2.JK
6R     R7R3.KL=R7R2.JK
```

```
6R     R8R3.KL=R8R2.JK
6R     R9R3.KL=R9R2.JK
6R     R5R4.KL=R5R3.JK                              DELAY 1.00 SEC
6R     R6R4.KL=R6R3.JK
6R     R7R4.KL=R7R3.JK
6R     R8R4.KL=R8R3.JK
6R     R9R4.KL=R9R3.JK
6R     R5R5.KL=R5R4.JK                              DELAY 1.25 SEC
6R     R6R5.KL=R6R4.JK
6R     R7R5.KL=R7R4.JK
6R     R8R5.KL=R8R4.JK
6R     R9R5.KL=R9R4.JK
6R     R5R6.KL=R5R5.JK                              DELAY 1.50 SEC
6R     R6R6.KL=R6R5.JK
6R     R7R6.KL=R7R5.JK
6R     R8R6.KL=R8R5.JK
6R     R9R6.KL=R9R5.JK
6R     R5R7.KL=R5R6.JK                              DELAY 1.75 SEC
6R     R6R7.KL=R6R6.JK
6R     R7R7.KL=R7R6.JK
6R     R8R7.KL=R8R6.JK
6R     R9R7.KL=R9R6.JK
NOTE
49A    A1.K=SWITCH(R1R5.JK,R1R2.JK,BLT0.K)                DELAYED RESPONSE
49A    A2.K=SWITCH(R2R5.JK,R2R2.JK,BLT1.K)                DELAYED RESPONSE
49A    A3.K=SWITCH(R3R5.JK,R3R2.JK,BLT2.K)                DELAYED RESPONSE
49A    A4.K=SWITCH(R4R5.JK,R4R2.JK,BLT3.K)                DELAYED RESPONSE
49A    A5.K=SWITCH(R5R5.JK,R5R2.JK,BLT4.K)                DELAYED RESPONSE
49A    A6.K=SWITCH(R6R5.JK,R6R2.JK,BLT5.K)                DELAYED RESPONSE
49A    A7.K=SWITCH(R7R5.JK,R7R2.JK,BLT6.K)                DELAYED RESPONSE
49A    A8.K=SWITCH(R8R5.JK,R8R2.JK,BLT7.K)                DELAYED RESPONSE
49A    A9.K=SWITCH(R9R5.JK,R9R2.JK,BLT8.K)                DELAYED RESPONSE
NOTE
54A    HA1.K=MIN(A1.K,MXA1.K)                             HYPO.ACCELERATION
54A    HA2.K=MIN(A2.K,MXA2.K)                             HYPO.ACCELERATION
54A    HA3.K=MIN(A3.K,MXA3.K)                             HYPO.ACCELERATION
54A    HA4.K=MIN(A4.K,MXA4.K)                             HYPO.ACCELERATION
54A    HA5.K=MIN(A5.K,MXA5.K)                             HYPO.ACCELERATION
54A    HA6.K=MIN(A6.K,MXA6.K)                             HYPO.ACCELERATION
54A    HA7.K=MIN(A7.K,MXA7.K)                             HYPO.ACCELERATION
54A    HA8.K=MIN(A8.K,MXA8.K)                             HYPO.ACCELERATION
54A    HA9.K=MIN(A9.K,MXA9.K)                             HYPO.ACCELERATION
56A    AB1.K=MAX(HA1.K,MXD)                         ACTUAL ACCELERATION
56A    AB2.K=MAX(HA2.K,MXD)                         ACTUAL ACCELERATION
56A    AB3.K=MAX(HA3.K,MXD)                         ACTUAL ACCELERATION
56A    AB4.K=MAX(HA4.K,MXD)                         ACTUAL ACCELERATION
56A    AB5.K=MAX(HA5.K,MXD)                         ACTUAL ACCELERATION
56A    AB6.K=MAX(HA6.K,MXD)                         ACTUAL ACCELERATION
56A    AB7.K=MAX(HA7.K,MXD)                         ACTUAL ACCELERATION
56A    AB8.K=MAX(HA8.K,MXD)                         ACTUAL ACCELERATION
56A    AB9.K=MAX(HA9.K,MXD)                         ACTUAL ACCELERATION
20A    GS1.K=-S1.K/DT                               BOUNDARY ACCELERATION RATE
20A    GS2.K=-S2.K/DT                               BOUNDARY ACCELERATION RATE
20A    GS3.K=-S3.K/DT                               BOUNDARY ACCELERATION RATE
20A    GS4.K=-S4.K/DT                               BOUNDARY ACCELERATION RATE
20A    GS5.K=-S5.K/DT                               BOUNDARY ACCELERATION RATE
20A    GS6.K=-S6.K/DT                               BOUNDARY ACCELERATION RATE
```

```
20A   GS7.K=-S7.K/DT                                BOUNDARY ACCELERATION RATE
20A   GS8.K=-S8.K/DT                                BOUNDARY ACCELERATION RATE
20A   GS9.K=-S9.K/DT                                BOUNDARY ACCELERATION RATE
51A   ABC1.K=CLIP(AB1.K,GS1.K,AB1.K,GS1.K)              REAL ACCELERATION
51A   ABC2.K=CLIP(AB2.K,GS2.K,AB2.K,GS2.K)              REAL ACCELERATION
51A   ABC3.K=CLIP(AB3.K,GS3.K,AB3.K,GS3.K)              REAL ACCELERATION
51A   ABC4.K=CLIP(AB4.K,GS4.K,AB4.K,GS4.K)              REAL ACCELERATION
51A   ABC5.K=CLIP(AB5.K,GS5.K,AB5.K,GS5.K)              REAL ACCELERATION
51A   ABC6.K=CLIP(AB6.K,GS6.K,AB6.K,GS6.K)              REAL ACCELERATION
51A   ABC7.K=CLIP(AB7.K,GS7.K,AB7.K,GS7.K)              REAL ACCELERATION
51A   ABC8.K=CLIP(AB8.K,GS8.K,AB8.K,GS8.K)              REAL ACCELERATION
51A   ABC9.K=CLIP(AB9.K,GS9.K,AB9.K,GS9.K)              REAL ACCELERATION
NOTE
NOTE     CONSIDERING ONLY ONE CAR AHAED
50A   RBV1.K=(SNS)(DS1.K)/(DL1.K-0)        RESPONSE TO DIF OF SPD PLUS
50A   RBV2.K=(SNS)(DS2.K)/(DL2.K-0)        RESPONSE TO DIF OF SPD PLUS
50A   RBV3.K=(SNS)(DS3.K)/(DL3.K-0)        RESPONSE TO DIF OF SPD PLUS
50A   RBV4.K=(SNS)(DS4.K)/(DL4.K-0)        RESPONSE TO DIF OF SPD PLUS
50A   RBV5.K=(SNS)(DS5.K)/(DL5.K-0)        RESPONSE TO DIF OF SPD PLUS
50A   RBV6.K=(SNS)(DS6.K)/(DL6.K-0)        RESPONSE TO DIF OF SPD PLUS
50A   RBV7.K=(SNS)(DS7.K)/(DL7.K-0)        RESPONSE TO DIF OF SPD PLUS
50A   RBV8.K=(SNS)(DS8.K)/(DL8.K-0)        RESPONSE TO DIF OF SPD PLUS
50A   RBV9.K=(SNS)(DS9.K)/(DL9.K-0)        RESPONSE TO DIF OF SPD PLUS
50A   RAV1.K=(SNS)(DS1.K)/(DL1.K-MMM)      RESPONSE TO DIF OF SPD MINUS
50A   RAV2.K=(SNS)(DS2.K)/(DL2.K-MMM)      RESPONSE TO DIF OF SPD MINUS
50A   RAV3.K=(SNS)(DS3.K)/(DL3.K-MMM)      RESPONSE TO DIF OF SPD MINUS
50A   RAV4.K=(SNS)(DS4.K)/(DL4.K-MMM)      RESPONSE TO DIF OF SPD MINUS
50A   RAV5.K=(SNS)(DS5.K)/(DL5.K-MMM)      RESPONSE TO DIF OF SPD MINUS
50A   RAV6.K=(SNS)(DS6.K)/(DL6.K-MMM)      RESPONSE TO DIF OF SPD MINUS
50A   RAV7.K=(SNS)(DS7.K)/(DL7.K-MMM)      RESPONSE TO DIF OF SPD MINUS
50A   RAV8.K=(SNS)(DS8.K)/(DL8.K-MMM)      RESPONSE TO DIF OF SPD
50A   RAV9.K=(SNS)(DS9.K)/(DL9.K-MMM)      RESPONSE TO DIF OF SPD
NOTE
51A   RPV1.K=CLIP(RBV1.K,RAV1.K,DS1.K,0)              RESPONSE TO SPD
51A   RPV2.K=CLIP(RBV2.K,RAV2.K,DS2.K,0)              RESPONSE TO SPD
51A   RPV3.K=CLIP(RBV3.K,RAV3.K,DS3.K,0)              RESPONSE TO SPD
51A   RPV4.K=CLIP(RBV4.K,RAV4.K,DS4.K,0)              RESPONSE TO SPD
51A   RPV5.K=CLIP(RBV5.K,RAV5.K,DS5.K,0)              RESPONSE TO SPD
51A   RPV6.K=CLIP(RBV6.K,RAV6.K,DS6.K,0)              RESPONSE TO SPD
51A   RPV7.K=CLIP(RBV7.K,RAV7.K,DS7.K,0)              RESPONSE TO SPD
51A   RPV8.K=CLIP(RBV8.K,RAV8.K,DS8.K,0)              RESPONSE TO SPD
51A   RPV9.K=CLIP(RBV9.K,RAV9.K,DS9.K,0)              RESPONSE TO SPD
NOTE
18A   RPL1.K=(SND)(DL1.K-DDL1.K)           RESPONSE FOR DISTANCE
18A   RPL2.K=(SND)(DL2.K-DDL2.K)           RESPONSE FOR DISTANCE
18A   RPL3.K=(SND)(DL3.K-DDL3.K)           RESPONSE FOR DISTANCE
18A   RPL4.K=(SND)(DL4.K-DDL4.K)           RESPONSE FOR DISTANCE
18A   RPL5.K=(SND)(DL5.K-DDL5.K)           RESPONSE FOR DISTANCE
18A   RPL6.K=(SND)(DL6.K-DDL6.K)           RESPONSE FOR DISTANCE
18A   RPL7.K=(SND)(DL7.K-DDL7.K)           RESPONSE FOR DISTANCE
18A   RPL8.K=(SND)(DL8.K-DDL8.K)           RESPONSE FOR DISTANCE
18A   RPL9.K=(SND)(DL9.K-DDL9.K)           RESPONSE FOR DISTANCE
NOTE
34A   NOS1.K=(1)NORMRN(0,ST1D)             ACC NOISE
34A   NOS2.K=(1)NORMRN(0,ST2D)             ACC NOISE
34A   NOS3.K=(1)NORMRN(0,ST3D)             ACC NOISE
34A   NOS4.K=(1)NORMRN(0,ST4D)             ACC NOISE
```

```
34A   NOS5.K=(1)NORMRN(0,ST5D)                    ACC NOISE
34A   NOS6.K=(1)NORMRN(0,ST6D)                    ACC NOISE
34A   NOS7.K=(1)NORMRN(0,ST7D)                    ACC NOISE
34A   NOS8.K=(1)NORMRN(0,ST8D)                    ACC NOISE
34A   NOS9.K=(1)NORMRN(0,ST9D)                    ACC NOISE
6N    ST1D=STDD
6N    ST2D=STDD
6N    ST3D=STDD
6N    ST4D=STDD
6N    ST5D=STDD
6N    ST6D=STDD
6N    ST7D=STDD
6N    ST8D=STDD
6N    ST9D=STDD
8A    RPS1.K=RPV1.K+RPL1.K+NOS1.K                 TOTAL RESPONSE
8A    RPS2.K=RPV2.K+RPL2.K+NOS2.K                 TOTAL RESPONSE
8A    RPS3.K=RPV3.K+RPL3.K+NOS3.K                 TOTAL RESPONSE
8A    RPS4.K=RPV4.K+RPL4.K+NOS4.K                 TOTAL RESPONSE
8A    RPS5.K=RPV5.K+RPL5.K+NOS5.K                 TOTAL RESPONSE
8A    RPS6.K=RPV6.K+RPL6.K+NOS6.K                 TOTAL RESPONSE
8A    RPS7.K=RPV7.K+RPL7.K+NOS7.K                 TOTAL RESPONSE
8A    RPS8.K=RPV8.K+RPL8.K+NOS8.K                 TOTAL RESPONSE
8A    RPS9.K=RPV9.K+RPL9.K+NOS9.K                 TOTAL RESPONSE
6A    SSA0.K=S0.K                                 SPEED OF CAR
51A   SSA1.K=CLIP(S1.K,0,MM1.K,NNN)               AFTER COLLISION SPEED
51A   SSA2.K=CLIP(S2.K,0,MM2.K,NNN)               AFTER COLLISION SPEED
51A   SSA3.K=CLIP(S3.K,0,MM3.K,NNN)               AFTER COLLISION SPEED
51A   SSA4.K=CLIP(S4.K,0,MM4.K,NNN)               AFTER COLLISION SPEED
51A   SSA5.K=CLIP(S5.K,0,MM5.K,NNN)               AFTER COLLISION SPEED
51A   SSA6.K=CLIP(S6.K,0,MM6.K,NNN)               AFTER COLLISION SPEED
51A   SSA7.K=CLIP(S7.K,0,MM7.K,NNN)               AFTER COLLISION SPEED
51A   SSA8.K=CLIP(S8.K,0,MM8.K,NNN)               AFTER COLLISION SPEED
51A   SSA9.K=CLIP(S9.K,0,MM9.K,NNN)               AFTER COLLISION SPEED
NOTE
NOTE      BOTTLENECK SPEED CONTROL
54R   SS0.KL=MIN(SSA0.K,MSL0.K)                   BOTTLENECK SPEED
54R   SS1.KL=MIN(SSA1.K,MSL1.K)                   BOTTLENECK SPEED
54R   SS2.KL=MIN(SSA2.K,MSL2.K)                   BOTTLENECK SPEED
54R   SS3.KL=MIN(SSA3.K,MSL3.K)                   BOTTLENECK SPEED
54R   SS4.KL=MIN(SSA4.K,MSL4.K)                   BOTTLENECK SPEED
54R   SS5.KL=MIN(SSA5.K,MSL5.K)                   BOTTLENECK SPEED
54R   SS6.KL=MIN(SSA6.K,MSL6.K)                   BOTTLENECK SPEED
54R   SS7.KL=MIN(SSA7.K,MSL7.K)                   BOTTLENECK SPEED
54R   SS8.KL=MIN(SSA8.K,MSL8.K)                   BOTTLENECK SPEED
54R   SS9.KL=MIN(SSA9.K,MSL9.K)                   BOTTLENECK SPEED
58A   MSL0.K=TABHL(TBNK,L0.K,0,1500,100)                MAX SPEED LIMIT
58A   MSL1.K=TABHL(TBNK,L1.K,0,1500,100)                MAX SPEED LIMIT
58A   MSL2.K=TABHL(TBNK,L2.K,0,1500,100)                MAX SPEED LIMIT
58A   MSL3.K=TABHL(TBNK,L3.K,0,1500,100)                MAX SPEED LIMIT
58A   MSL4.K=TABHL(TBNK,L4.K,0,1500,100)                MAX SPEED LIMIT
58A   MSL5.K=TABHL(TBNK,L5.K,0,1500,100)                MAX SPEED LIMIT
58A   MSL6.K=TABHL(TBNK,L6.K,0,1500,100)                MAX SPEED LIMIT
58A   MSL7.K=TABHL(TBNK,L7.K,0,1500,100)                MAX SPEED LIMIT
58A   MSL8.K=TABHL(TBNK,L8.K,0,1500,100)                MAX SPEED LIMIT
58A   MSL9.K=TABHL(TBNK,L9.K,0,1500,100)                MAX SPEED LIMIT
NOTE
14A   DDL1.K=20+(SLOP)(SSA1.K)                    DESIRED DISTANCE
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14A   DDL2.K=20+(SLOP)(SSA2.K)          DESIRED DISTANCE
14A   DDL3.K=20+(SLOP)(SSA3.K)          DESIRED DISTANCE
14A   DDL4.K=20+(SLOP)(SSA4.K)          DESIRED DISTANCE
14A   DDL5.K=20+(SLOP)(SSA5.K)          DESIRED DISTANCE
14A   DDL6.K=20+(SLOP)(SSA6.K)          DESIRED DISTANCE
14A   DDL7.K=20+(SLOP)(SSA7.K)          DESIRED DISTANCE
14A   DDL8.K=20+(SLOP)(SSA8.K)          DESIRED DISTANCE
14A   DDL9.K=20+(SLOP)(SSA9.K)          DESIRED DISTANCE
NOTE
7A    DS1.K=SSA0.K-SSA1.K               DIFFERENCE OF SPEED
7A    DS2.K=SSA1.K-SSA2.K               DIFFERENCE OF SPEED
7A    DS3.K=SSA2.K-SSA3.K               DIFFERENCE OF SPEED
7A    DS4.K=SSA3.K-SSA4.K               DIFFERENCE OF SPEED
7A    DS5.K=SSA4.K-SSA5.K               DIFFERENCE OF SPEED
7A    DS6.K=SSA5.K-SSA6.K               DIFFERENCE OF SPEED
7A    DS7.K=SSA6.K-SSA7.K               DIFFERENCE OF SPEED
7A    DS8.K=SSA7.K-SSA8.K               DIFFERENCE OF SPEED
7A    DS9.K=SSA8.K-SSA9.K               DIFFERENCE OF SPEED
7A    LL1.K=L0.K-L1.K                   DISTANCE
7A    LL2.K=L1.K-L2.K                   DISTANCE
7A    LL3.K=L2.K-L3.K                   DISTANCE
7A    LL4.K=L3.K-L4.K                   DISTANCE
7A    LL5.K=L4.K-L5.K                   DISTANCE
7A    LL6.K=L5.K-L6.K                   DISTANCE
7A    LL7.K=L6.K-L7.K                   DISTANCE
7A    LL8.K=L7.K-L8.K                   DISTANCE
7A    LL9.K=L8.K-L9.K                   DISTANCE
54A   MM1.K=MIN(LL1.K,LL2.K)            HIT OR HITTED
54A   MM2.K=MIN(LL2.K,LL3.K)            HIT OR HITTED
54A   MM3.K=MIN(LL3.K,LL4.K)            HIT OR HITTED
54A   MM4.K=MIN(LL4.K,LL5.K)            HIT OR HITTED
54A   MM5.K=MIN(LL5.K,LL6.K)            HIT OR HITTED
54A   MM6.K=MIN(LL6.K,LL7.K)            HIT OR HITTED
54A   MM7.K=MIN(LL7.K,LL8.K)            HIT OR HITTED
54A   MM8.K=MIN(LL8.K,LL9.K)            HIT OR HITTED
6A    MM9.K=LL9.K
56A   DL1.K=MAX(5,LL1.K)                MIN DISTANCE
56A   DL2.K=MAX(5,LL2.K)                MIN DISTANCE
56A   DL3.K=MAX(5,LL3.K)                MIN DISTANCE
56A   DL4.K=MAX(5,LL4.K)                MIN DISTANCE
56A   DL5.K=MAX(5,LL5.K)                MIN DISTANCE
56A   DL6.K=MAX(5,LL6.K)                MIN DISTANCE
56A   DL7.K=MAX(5,LL7.K)                MIN DISTANCE
56A   DL8.K=MAX(5,LL8.K)                MIN DISTANCE
56A   DL9.K=MAX(5,LL9.K)                MIN DISTANCE
NOTE
NOTE     LIMITS
NOTE
18A   MXA1.K=(0.07)(MXS1-SSA1.K)        MAXIMUM ACCELERATION
18A   MXA2.K=(0.07)(MXS1-SSA2.K)        MAXIMUM ACCELERATION
18A   MXA3.K=(0.07)(MXS1-SSA3.K)        MAXIMUM ACCELERATION
18A   MXA4.K=(0.07)(MXS1-SSA4.K)        MAXIMUM ACCELERATION
18A   MXA5.K=(0.07)(MXS1-SSA5.K)        MAXIMUM ACCELERATION
18A   MXA6.K=(0.07)(MXS1-SSA6.K)        MAXIMUM ACCELERATION
18A   MXA7.K=(0.07)(MXS1-SSA7.K)        MAXIMUM ACCELERATION
18A   MXA8.K=(0.07)(MXS1-SSA8.K)        MAXIMUM ACCELERATION
18A   MXA9.K=(0.07)(MXS1-SSA9.K)        MAXIMUM ACCELERATION
```

```
NOTE
51R    AA1.KL=CLIP(ABC1.K,0,DL1.K,MMM)   AFTER COLLISION ACCELERATION
51R    AA2.KL=CLIP(ABC2.K,0,DL2.K,MMM)   AFTER COLLISION ACCELERATION
51R    AA3.KL=CLIP(ABC3.K,0,DL3.K,MMM)   AFTER COLLISION ACCELERATION
51R    AA4.KL=CLIP(ABC4.K,0,DL4.K,MMM)   AFTER COLLISION ACCELERATION
51R    AA5.KL=CLIP(ABC5.K,0,DL5.K,MMM)   AFTER COLLISION ACCELERATION
51R    AA6.KL=CLIP(ABC6.K,0,DL6.K,MMM)   AFTER COLLISION ACCELERATION
51R    AA7.KL=CLIP(ABC7.K,0,DL7.K,MMM)   AFTER COLLISION ACCELERATION
51R    AA8.KL=CLIP(ABC8.K,0,DL8.K,MMM)   AFTER COLLISION ACCELERATION
51R    AA9.KL=CLIP(ABC9.K,0,DL9.K,MMM)   AFTER COLLISION ACCELERATION
NOTE
NOTE      EFFECT OF BRAKE LIGHTS
NOTE
12A    CDC0.K=(0,035)(SSA0.K))           COASTING DECELERATION
12A    CDC1.K=(0,035)(SSA1.K))           COASTING DECELERATION
12A    CDC2.K=(0,035)(SSA2.K))           COASTING DECELERATION
12A    CDC3.K=(0,035)(SSA3.K))           COASTING DECELERATION
12A    CDC4.K=(0,035)(SSA4.K))           COASTING DECELERATION
12A    CDC5.K=(0,035)(SSA5.K))           COASTING DECELERATION
12A    CDC6.K=(0,035)(SSA6.K))           COASTING DECELERATION
12A    CDC7.K=(0,035)(SSA7.K))           COASTING DECELERATION
12A    CDC8.K=(0,035)(SSA8.K))           COASTING DECELERATION
12A    CDC9.K=(0,035)(SSA9.K))           COASTING DECELERATION
51A    BLT0.K=CLIP(1,0,-CDC0.K,AA0.JK)   BRAKE LIGHT
51A    BLT1.K=CLIP(1,0,-CDC1.K,AA1.JK)   BRAKE LIGHT
51A    BLT2.K=CLIP(1,0,-CDC2.K,AA2.JK)   BRAKE LIGHT
51A    BLT3.K=CLIP(1,0,-CDC3.K,AA3.JK)   BRAKE LIGHT
51A    BLT4.K=CLIP(1,0,-CDC4.K,AA4.JK)   BRAKE LIGHT
51A    BLT5.K=CLIP(1,0,-CDC5.K,AA5.JK)   BRAKE LIGHT
51A    BLT6.K=CLIP(1,0,-CDC6.K,AA6.JK)   BRAKE LIGHT
51A    BLT7.K=CLIP(1,0,-CDC7.K,AA7.JK)   BRAKE LIGHT
51A    BLT8.K=CLIP(1,0,-CDC8.K,AA8.JK)   BRAKE LIGHT
51A    BLT9.K=CLIP(1,0,-CDC9.K,AA9.JK)   BRAKE LIGHT
NOTE
NOTE      MANEUVER OF  THE FIRST CAR
NOTE
34R    AA0.KL=(1)NORMRN(0,STOD)
NOTE
NOTE      INITIAL VALUE
NOTE
7N     S0=SPD+NSS0
7N     S1=SPD+NSS1
7N     S2=SPD+NSS2
7N     S3=SPD+NSS3
7N     S4=SPD+NSS4
7N     S5=SPD+NSS5
7N     S6=SPD+NSS6
7N     S7=SPD+NSS7
7N     S8=SPD+NSS8
7N     S9=SPD+NSS9
C      NSS0=0.4
C      NSS1=0.4
C      NSS2=0.1
C      NSS3=0.1
C      NSS4=-2.1
C      NSS5=-1.1
C      NSS6=2.7
```

```
C       NSS7=1.6
C       NSS8=-0.3
C       NSS9=-1.6
14N     L0=-NSL0+(-0)(DST)
14N     L1=-NSL1+(-1)(DST)
14N     L2=-NSL2+(-2)(DST)
14N     L3=-NSL3+(-3)(DST)
14N     L4=-NSL4+(-4)(DST)
14N     L5=-NSL5+(-5)(DST)
14N     L6=-NSL6+(-6)(DST)
14N     L7=-NSL7+(-7)(DST)
14N     L8=-NSL8+(-8)(DST)
14N     L9=-NSL9+(-9)(DST)
C       NSL0=0
C       NSL1=0
C       NSL2=1.0
C       NSL3=2.5
C       NSL4=2.6
C       NSL5=-3.4
C       NSL6=-0.3
C       NSL7=7.0
C       NSL8=9.3
C       NSL9=9.2
NOTE
NOTE       CONSTANTS
NOTE
C       NNN=15          FT          COLLISION DISTANCE
C       MMM=15          FT          MINIMUM DISTANCE
C       SNS=40
C       SND=0.065
C       SLOP=1.2
C       SPD=80
C       DST=116
C       MXD=-20
C       MXS1=140
C       STOD=0.32                   FT/S**-2  ACC NOISE
C       STDD=0.32                   FT/S**-2  ACC NOISE
C       TBNK*=80/76/72/68/64/60/56/52/48/44/40/40/50/60/70/80
PLOT    SS0=0,SS1=1,SS2=2,SS3=3,SS4=4,SS5=5,SS6=6,SS7=7,SS8=8,SS9=9(0,100)
X1
SPEC    DT=0.25/LENGTH=50/PRTPER=0.25/PLTPER=0.5
RUN     FUJI-2
C       TBNK*=80/80/80/80/74/68/63/57/52/46/40/40/50/60/70/80
RUN     FUJI-3
C       TBNK*=80/80/80/80/80/80/80/70/60/50/40/40/50/60/70/80
RUN     FUJI-4
C       TBNK*=80/80/80/80/80/80/80/72/64/53/40/40/50/60/70/80
END

NORMAL EXIT.  EXECUTION TIME:         17565 MLSEC.
@FOR,IN              DATA
FOR S11F-03/27/75-17:44:51 (,0)

   STORAGE USED: CODE(1) 000000; DATA(0) 000001; BLANK COMMON(2) 000000
```

APPENDIX C

TYPICAL RUN RESULT

Two types of output are available in DYNAMO; one is plot output which is useful to understand whole situations and relationship between various factors graphically; the other is tabular form output which is prepared to know more details and accuracy by means of figures.

In this study above both types of output was used, especially tabular output which shows every changing of such important terms as location, speed, acceleration, headway, desired headway, etc. by every 0.25 second, was used to know details of collision time and speed.

There are too many volumes to attach the whole output. A few typical plot outputs are shown here. In the output the vertical axis is always time, and horizontal axis is headway speed on location which depends on the plotted factor; the figures in plot correspond to the car number.

List of Runs

Symbol	Description	Output	Condition
DS-7.1	Delay Type Selection	Headway	First Order Delay, $T = 15$, $c_1 = 1.047$
DS-7.2	Delay Type Selection	Speed, etc.	First Order Delay, $T = 15$, $c_1 = 1.047$
DS-15.1	Delay Type Selection	Headway	Third Order Delay, $T = 15$, $c_1 = 1.047$
DS-15.2	Delay Type Selection	Speed, etc.	Third Order Delay, $T = 15$, $c_1 = 1.047$
DS-23.1	Delay Type Selection	Headway	Pipeline Delay, $T = 15$, $c_1 = 1.047$
DS-23.2	Delay Type Selection	Speed, etc.	Pipeline Delay, $T = 15$, $c_1 = 1.047$
NS-1	Acceleration Noise	Speed	$\sigma D = 0.32$, IS = 75, ID = 110
NS-2	Acceleration Noise	Speed	$\sigma D = 0.32$, IS = 25, ID = 50
NS-3	Acceleration Noise	Speed	$\sigma D = 0.32$, IS, ID = 20s of NS-1
ES-1	Emergency Stop	Location	$c_2=30$, $c_5=.05$, $T_2=.5$, b=.8, IS=45, MD=15
ES-2	Emergency Stop	Location	$c_2=50$, $c_5=.05$, $T_2=.5$, b=.8, IS=75, MD=25
ES-4	Emergency Stop	Location	$c_2=50$; $c_5=.05$, $T_2=.5$, b=.8, IS=45, MD=25
NSRS-1	Noise Added Stop-Run-Stop	Location	$c_2=30$, $c_5=.05$, $T_1=1.0$, $T_2=.5$, b=.8, MD=15, MS=105
NSRS-9	Noise Added Stop-Run-Stop	Location	$c_2=30$, $c_5=.05$, $T_1=1.0$, $T_2=1.0$, b=1.5, MD=25 MS=175
MRG-1	Merging	Location	MFS=50, GAP=2, ACC=T, RS=20
MRG-2	Merging	Location	MFS=80, GAP=2, ACC=T, RS=20
MRG-3	Merging	Location	MFS=50, GAP=4, ACC=T, RS=20
BTN-1	Flow in Bottleneck	Speed	Bottleneck Shape 1
BTN-2	Flow in Bottleneck	Speed	Bottleneck Shape 2
BTN-3	Flow in Bottleneck	Speed	Bottleneck Shape 3

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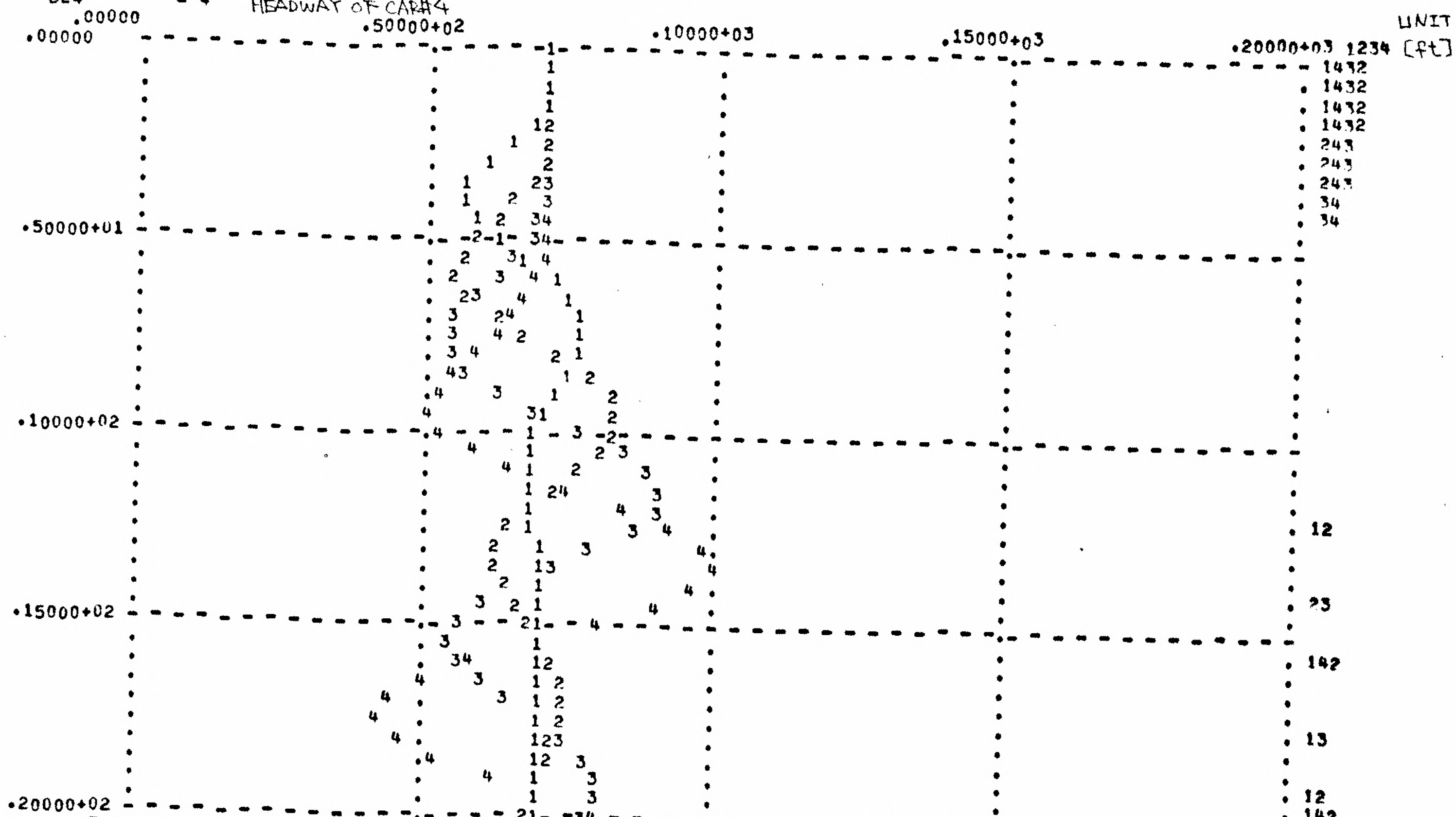
RUN DS-7.1

Distance between cars

DL1 = 1 HEADWAY OF CAR#1
DL2 = 2 HEADWAY OF CAR#2
DL3 = 3 HEADWAY OF CAR#3
DL4 = 4 HEADWAY OF CAR#4

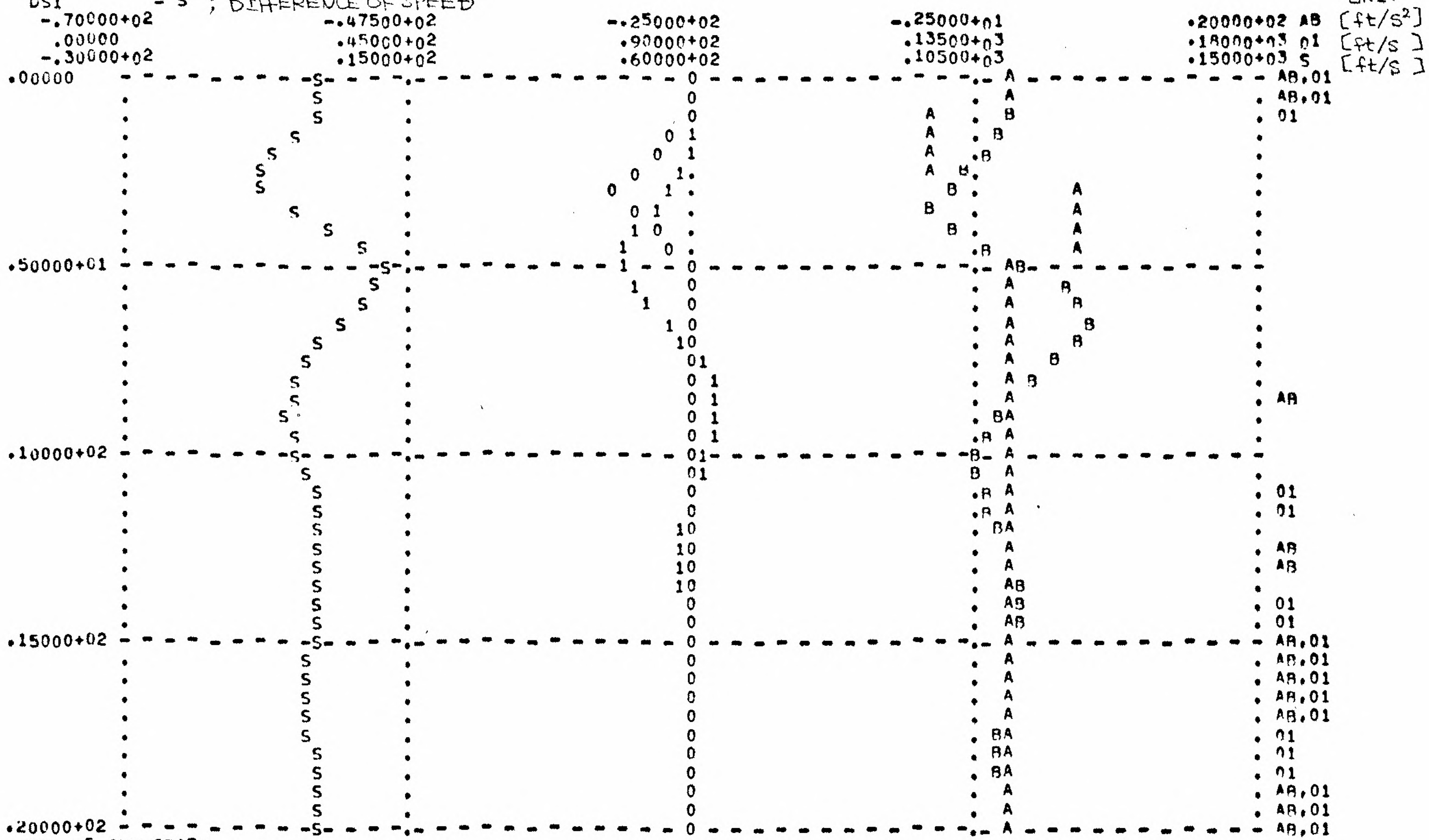


END OF ONE GRAPH

Acceleration and Speed

AA0 = A ; ACCELERATION OF CAR#0
AA1 = B ; ACCELERATION OF CAR#1
SS0 = 0 ; SPEED OF CAR#0
SS1 = 1 ; SPEED OF CAR#1
DS1 = S ; DIFFERENCE OF SPEED

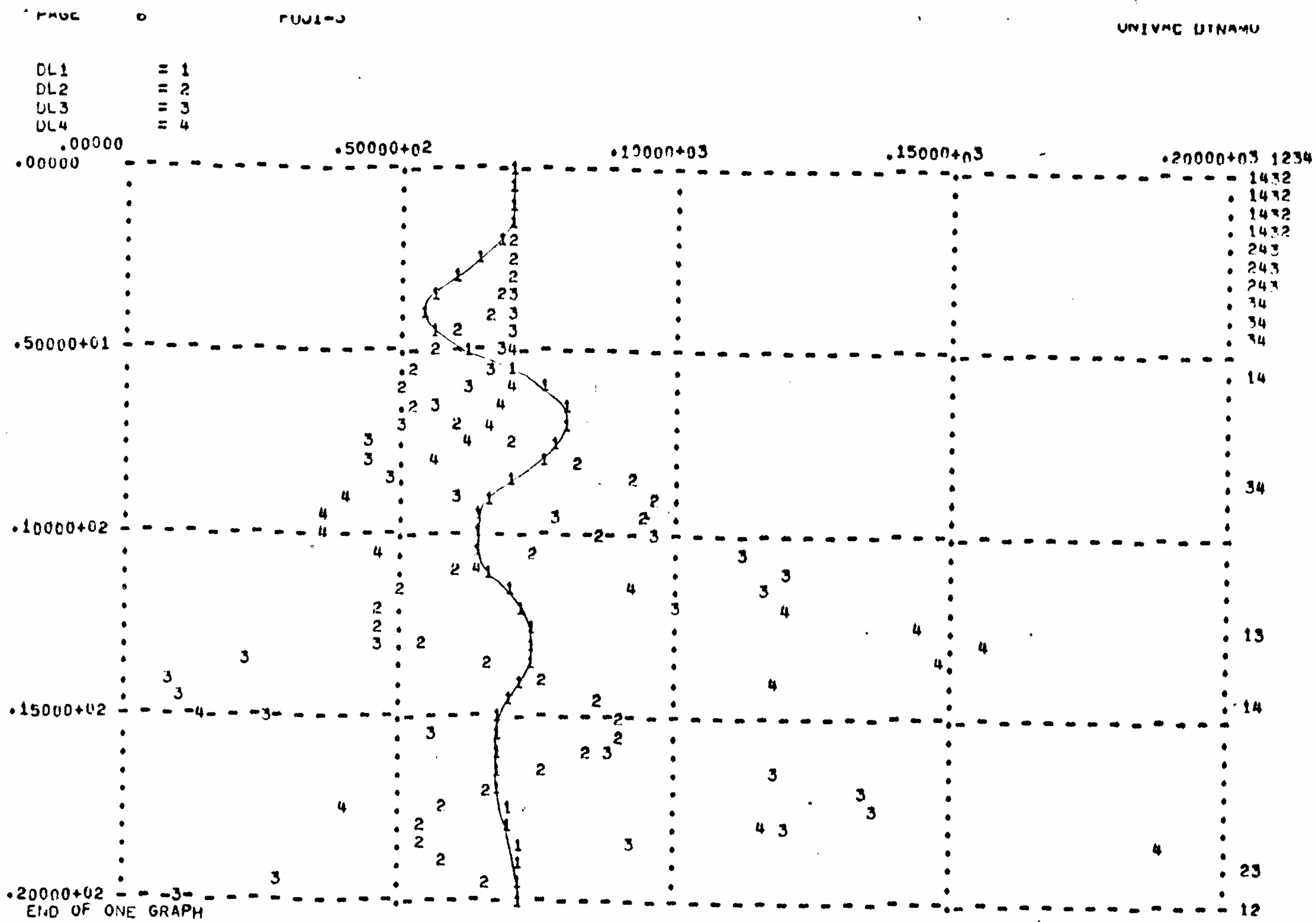
RUN DS-7.2



END OF ONE GRAPH

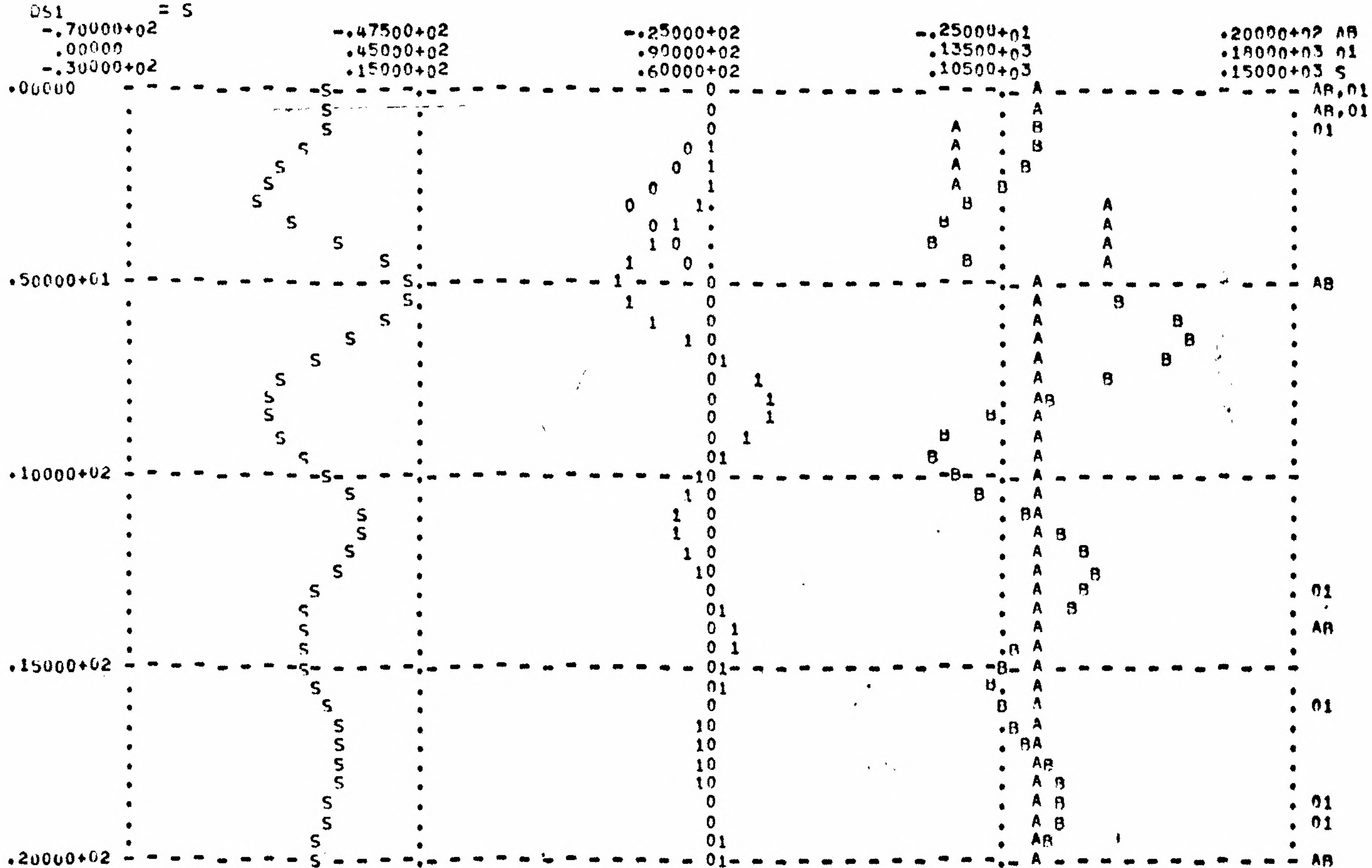
NORMAL EXIT. EXECUTION TIME: 3910 MLSEC.
QHDG,N X.M,66,6,3

RUN DS-15.1


PAGE 6 FOO1-3
UNIVAC DYNAMO
DL1 = 1
DL2 = 2
DL3 = 3
DL4 = 4
.00000
.50000+02
.10000+03
.15000+03
.20000+03 1234
.00000
.50000+01
.10000+02
.15000+02
.20000+02
END OF ONE GRAPH

RUN DS-15.2

AA0 = A
AA1 = B
SS0 = 0
SS1 = 1
DS1 = S

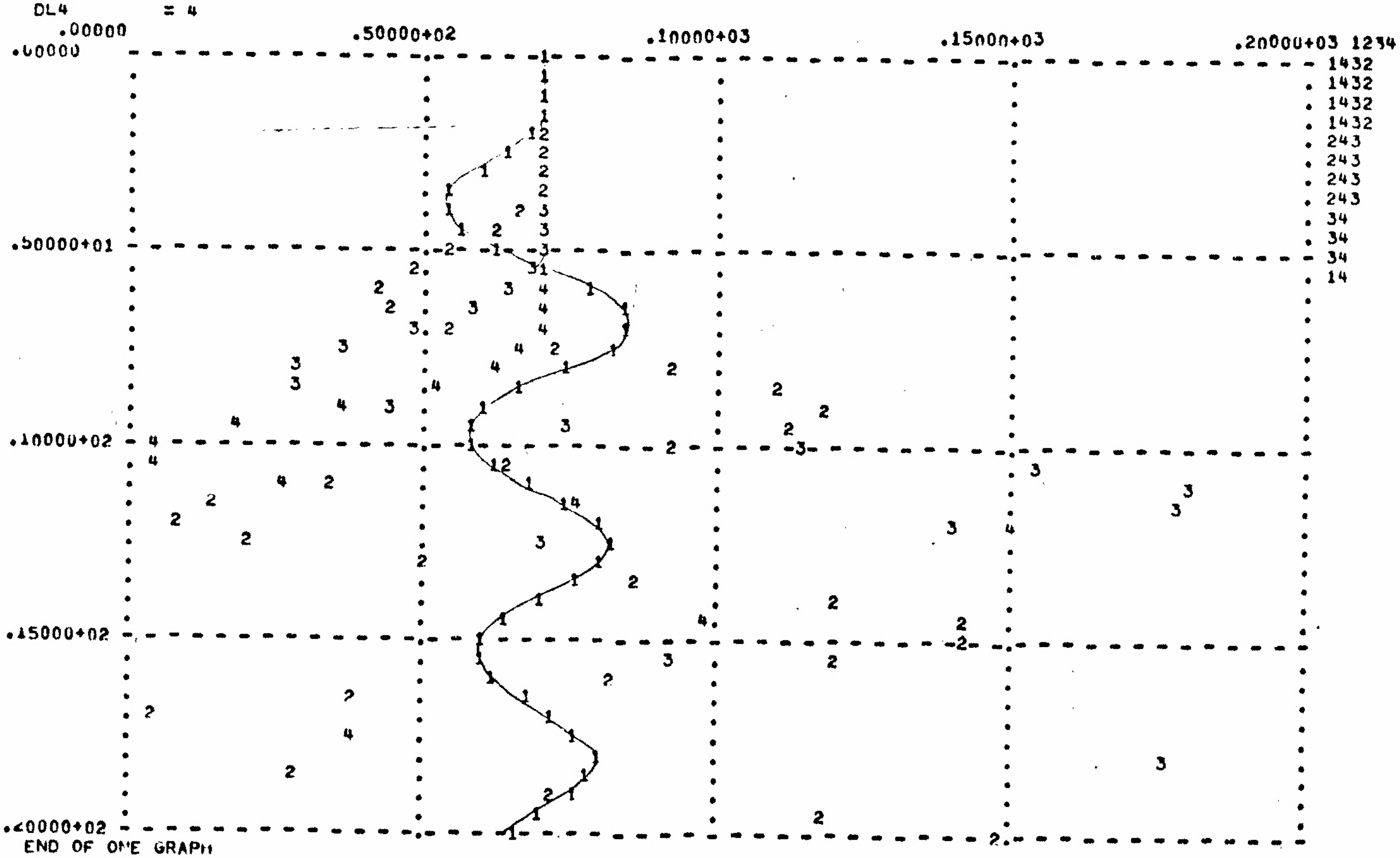


END OF ONE GRAPH

NORMAL EXIT. EXECUTION TIME: 3771 MLSEC.
SHDG,N X,M,66,6,3

D3
RT=1.5
SNS=1.047

DL1 = 1
DL2 = 2
DL3 = 3
DL4 = 4

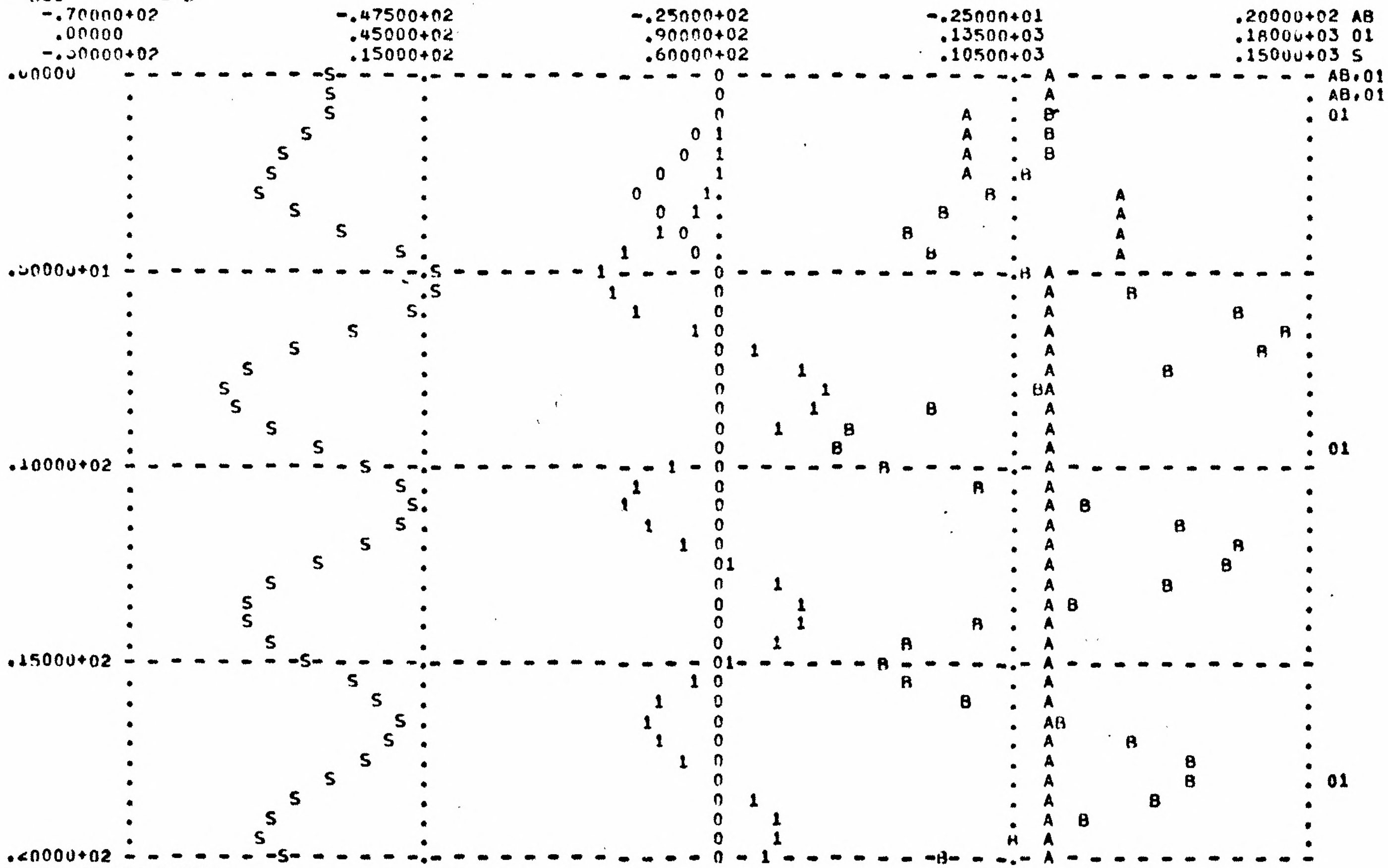


PD
RT=1.5
SNS=1.047

RUN DS-23.1

PAGE 7 FOOJ-3 UNIVAC DYNAMO

AA0 = A
AA1 = B
SS0 = 0
SS1 = 1
DS1 = S



END OF ONE GRAPH

NORMAL EXIT. EXECUTION TIME: 3395 MLSEC.
QHDG,N X.4.66.6.3

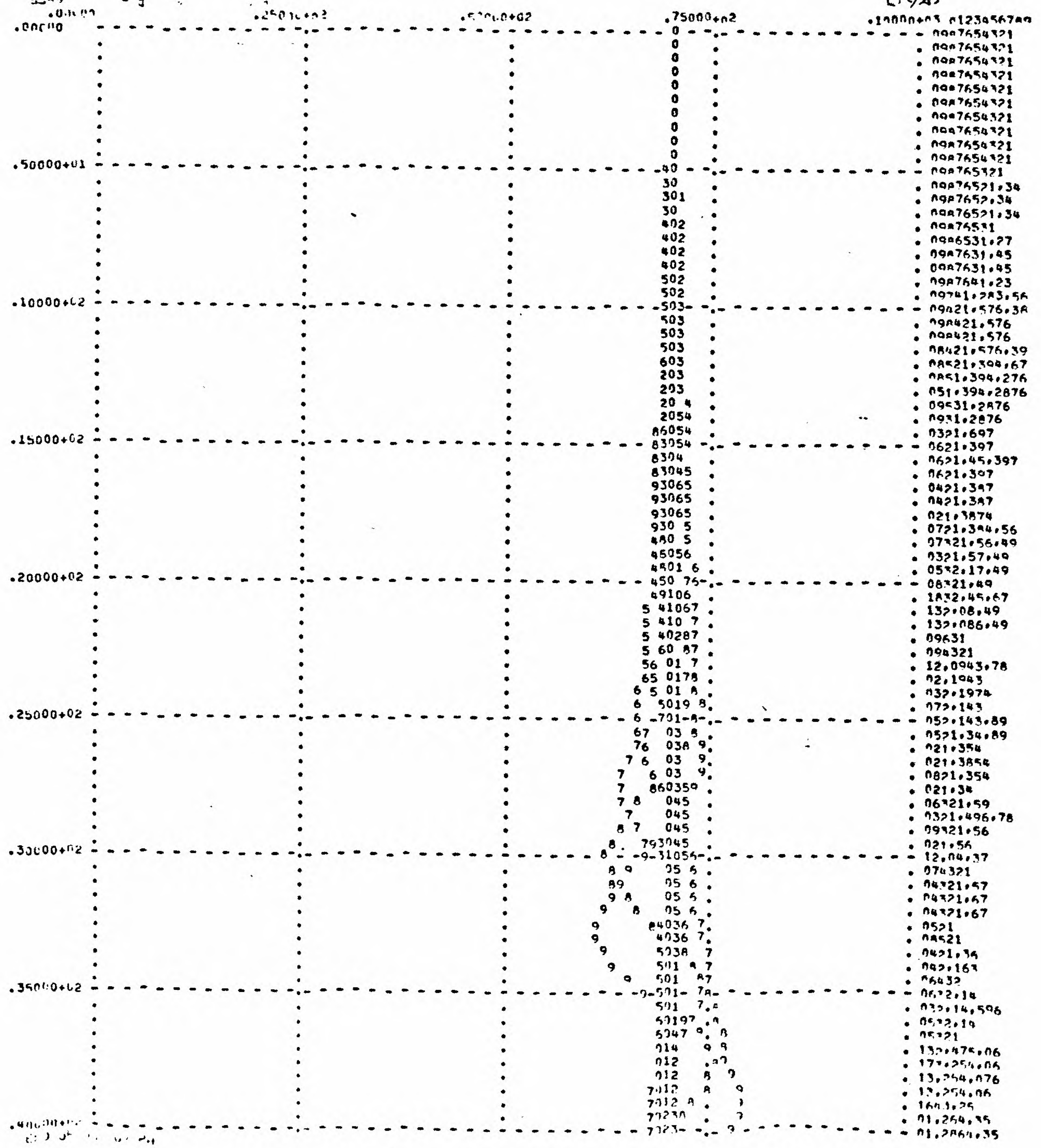
PD
RT=1.5
SNS=1.047

RUN DS-23.2

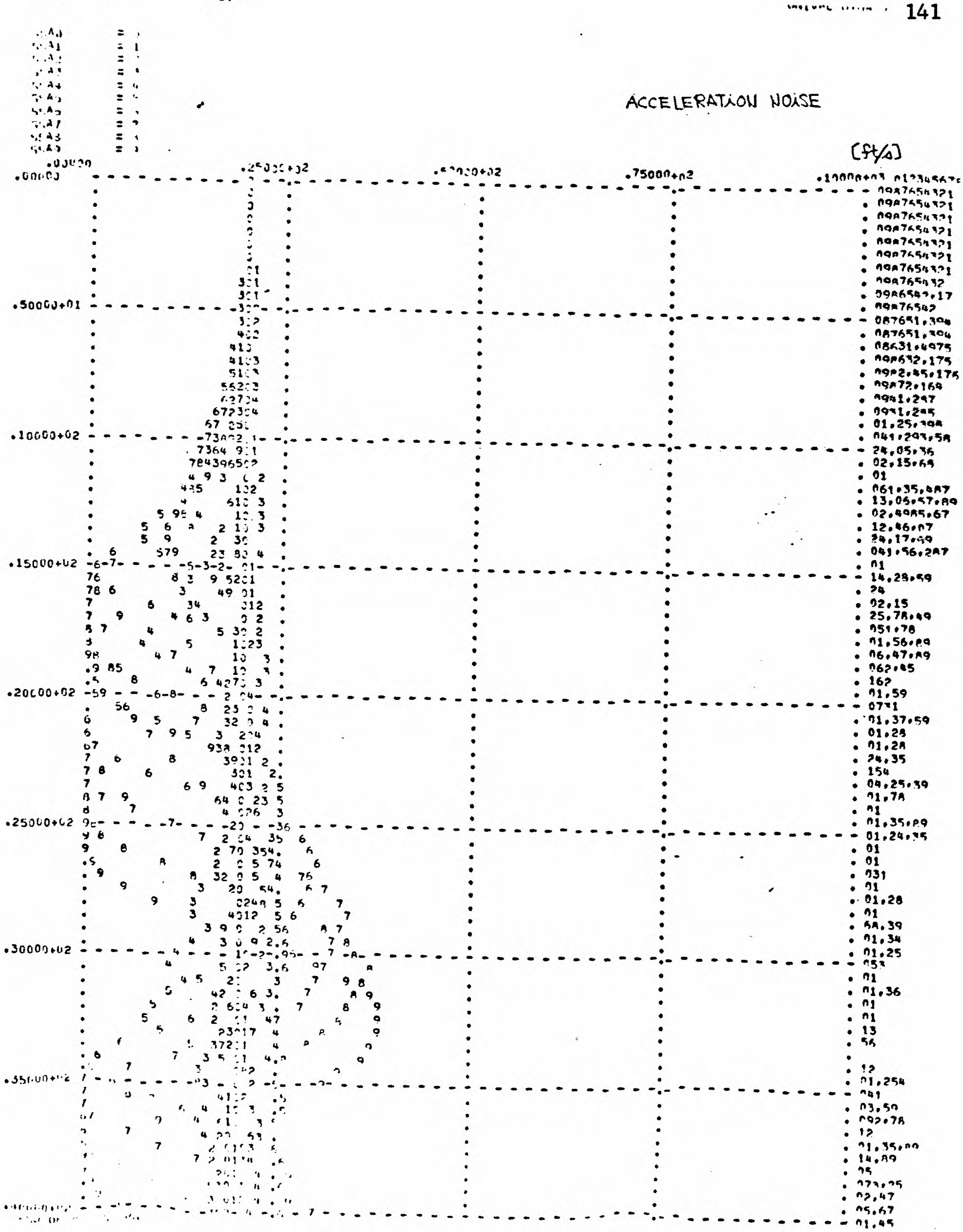
Acceleration Noise

SSA0 = 0 SPEED OF CARRO
SSA1 = 1
SSA2 = 2
SSA3 = 3
SSA4 = 4
SSA5 = 5
SSA6 = 6
SSA7 = 7
SSA8 = 8
SSA9 = 9

[ft/s]



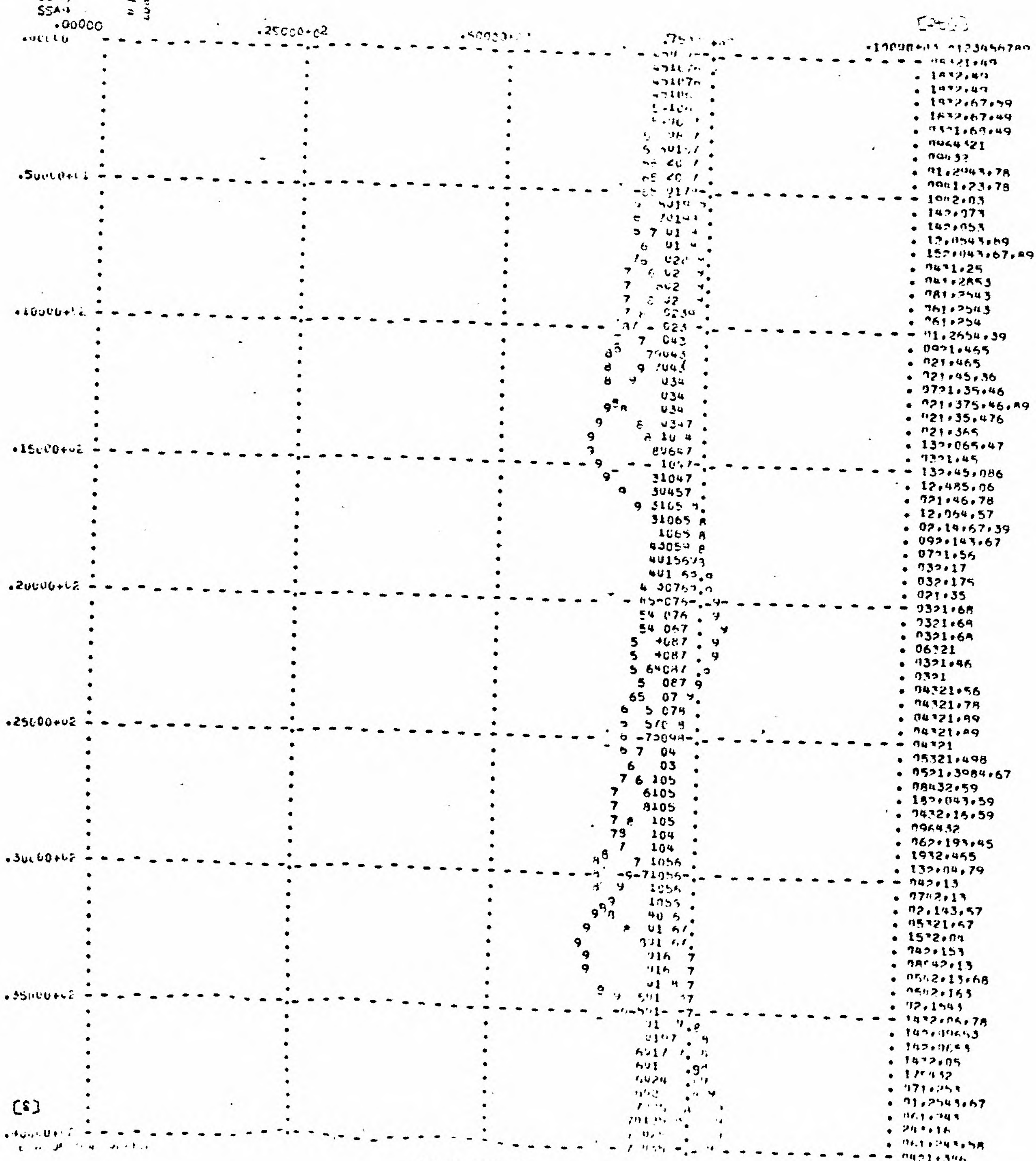
RUN NS-1


ACCELERATION NOISE
(ft/s)
.00000
.50000+01
.10000+02
.15000+02
.20000+02
.25000+02
.30000+02
.35000+02
.75000+02

RUN NS-2

SSA0 = 0
SSA1 = 1
SSA2 = 2
SSA3 = 3
SSA4 = 4
SSA5 = 5
SSA6 = 6
SSA7 = 7
SSA8 = 8
SSA9 = 9

ACCELERATION NOISE



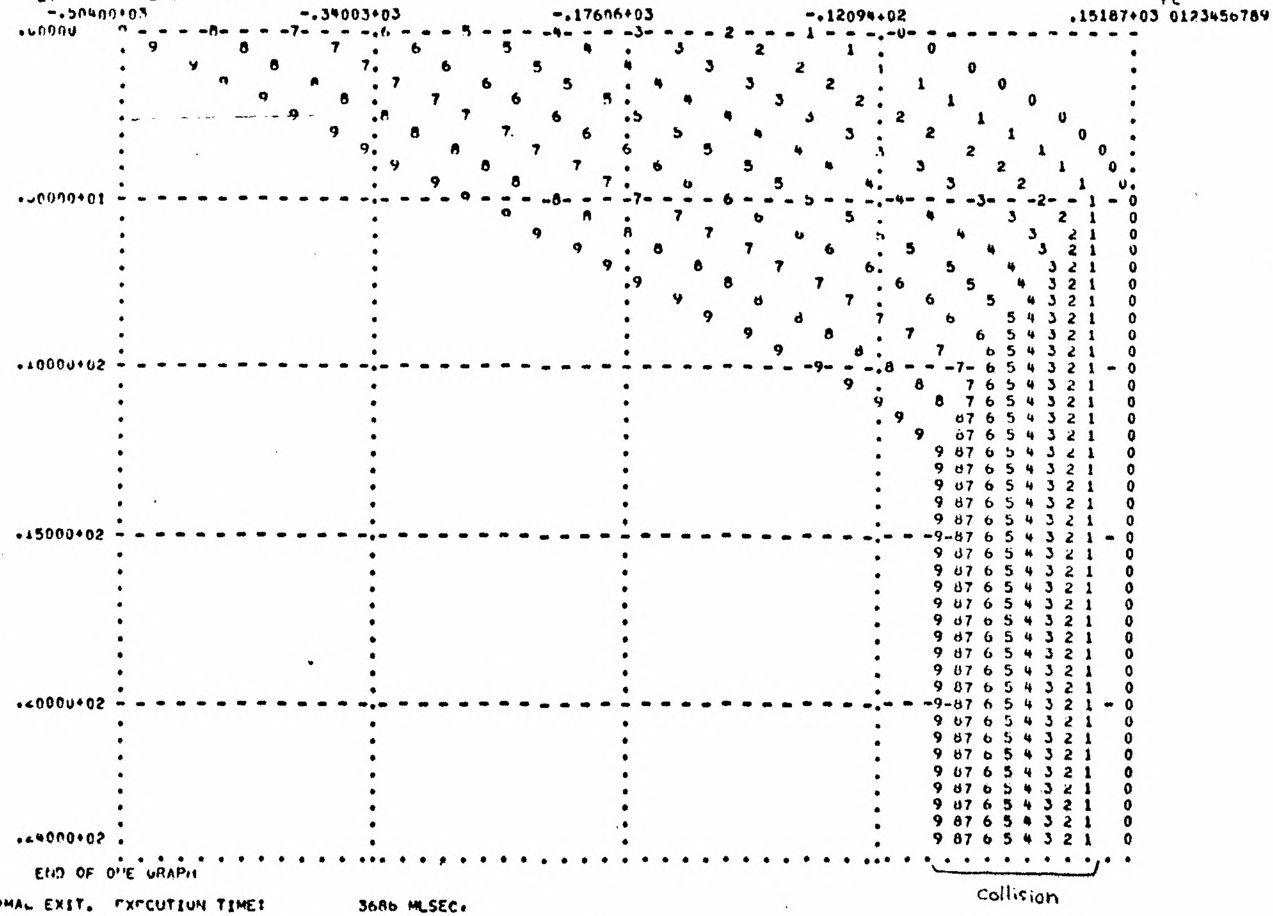
RUN NS-3

PAGE 7 FUJI-1 UNIVAC DYNAMO

ES-1

RUN ES-1

L0 = 0 LOCATION OF CAR NO
L1 = 1 " " 1
L2 = 2 " " 2
L3 = 3 " " 3
L4 = 4 " " 4
L5 = 5 " " 5
L6 = 6 " " 6
L7 = 7 " " 7
L8 = 8 " " 8
L9 = 9 " " 9



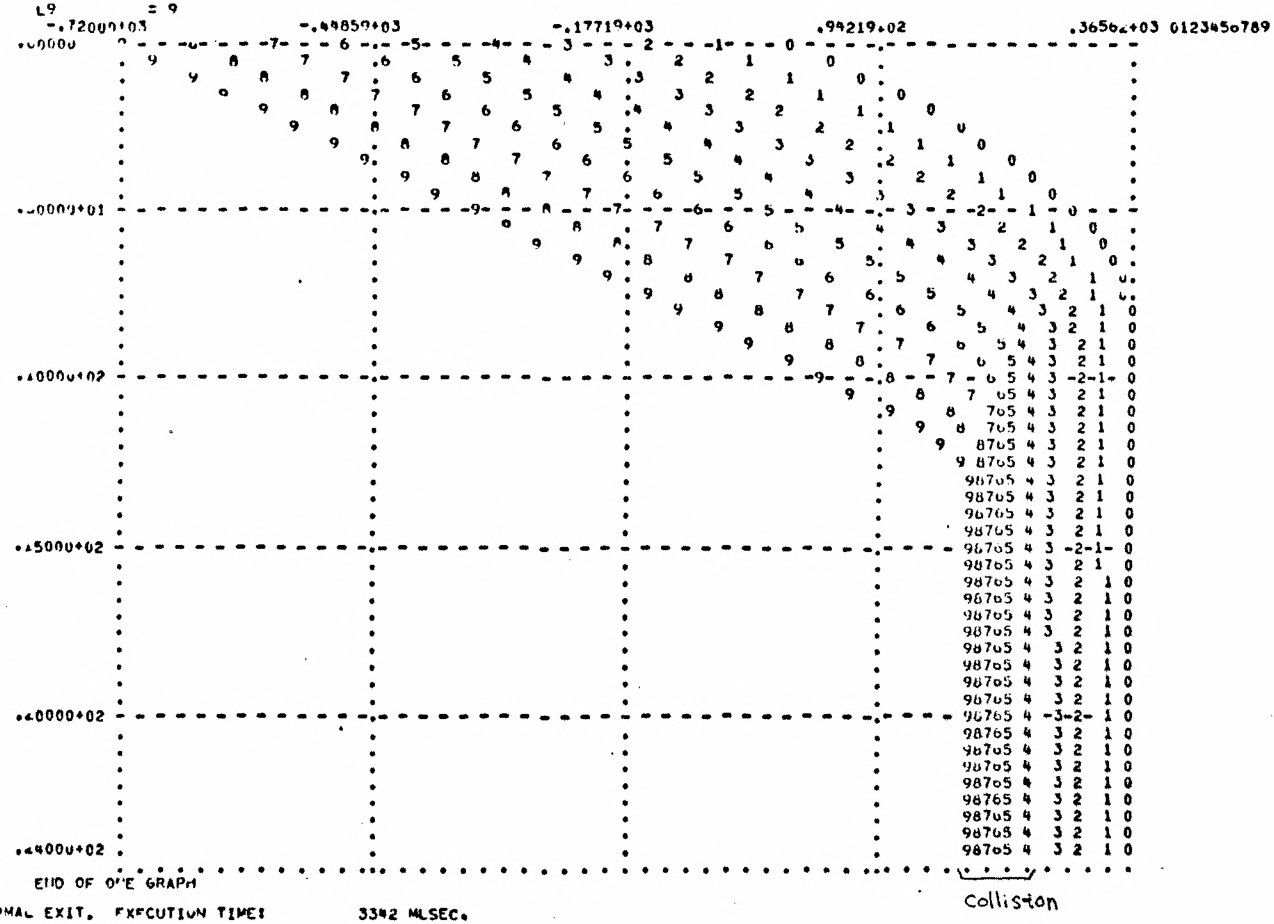
END OF ONE GRAPH

NORMAL EXIT. EXECUTION TIME: 3686 MLSEC.

ES-2

RUN ES-2

L0 = 0
L1 = 1
L2 = 2
L3 = 3
L4 = 4
L5 = 5
L6 = 6
L7 = 7
L8 = 8
L9 = 9



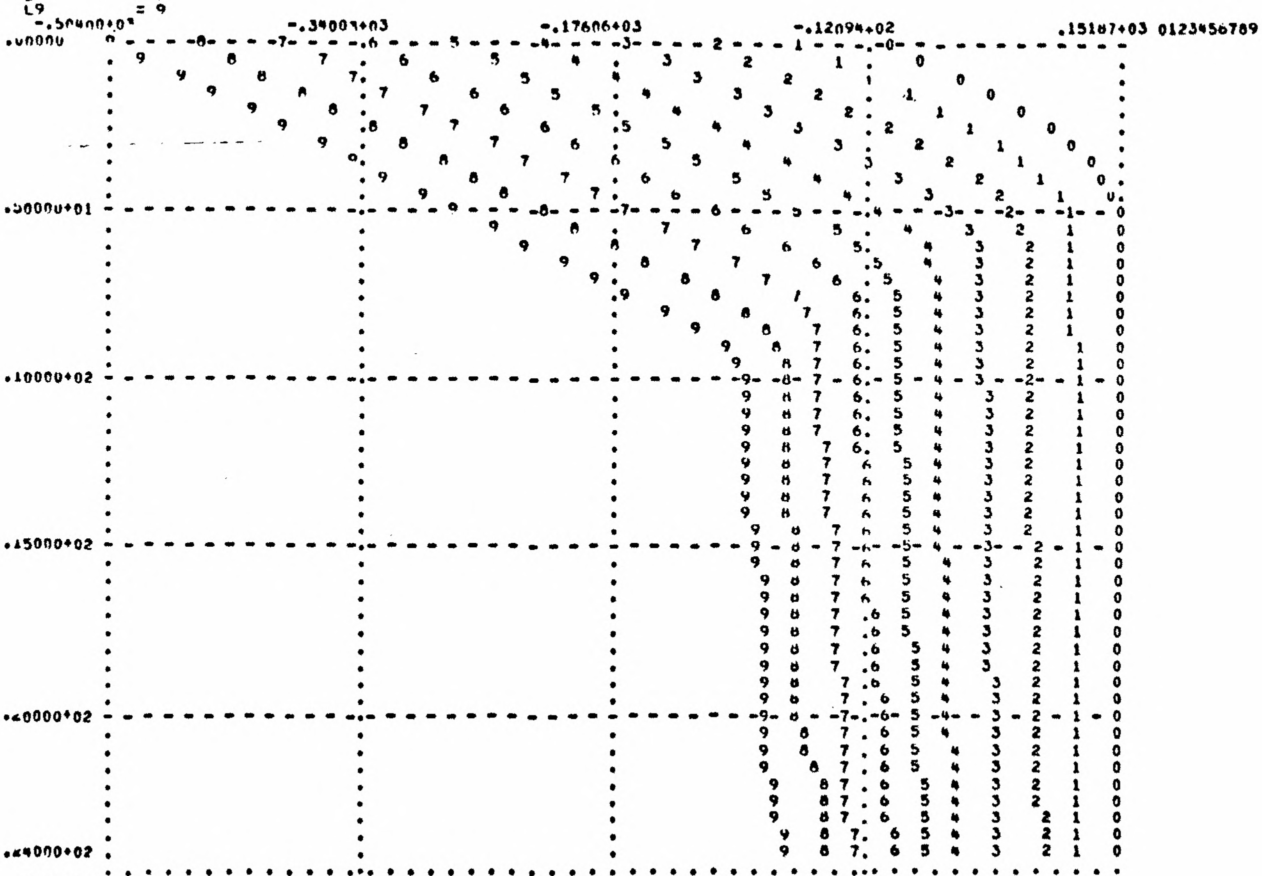
END OF ONE GRAPH

NORMAL EXIT. EXECUTION TIME: 3342 MLSEC.

ES-4

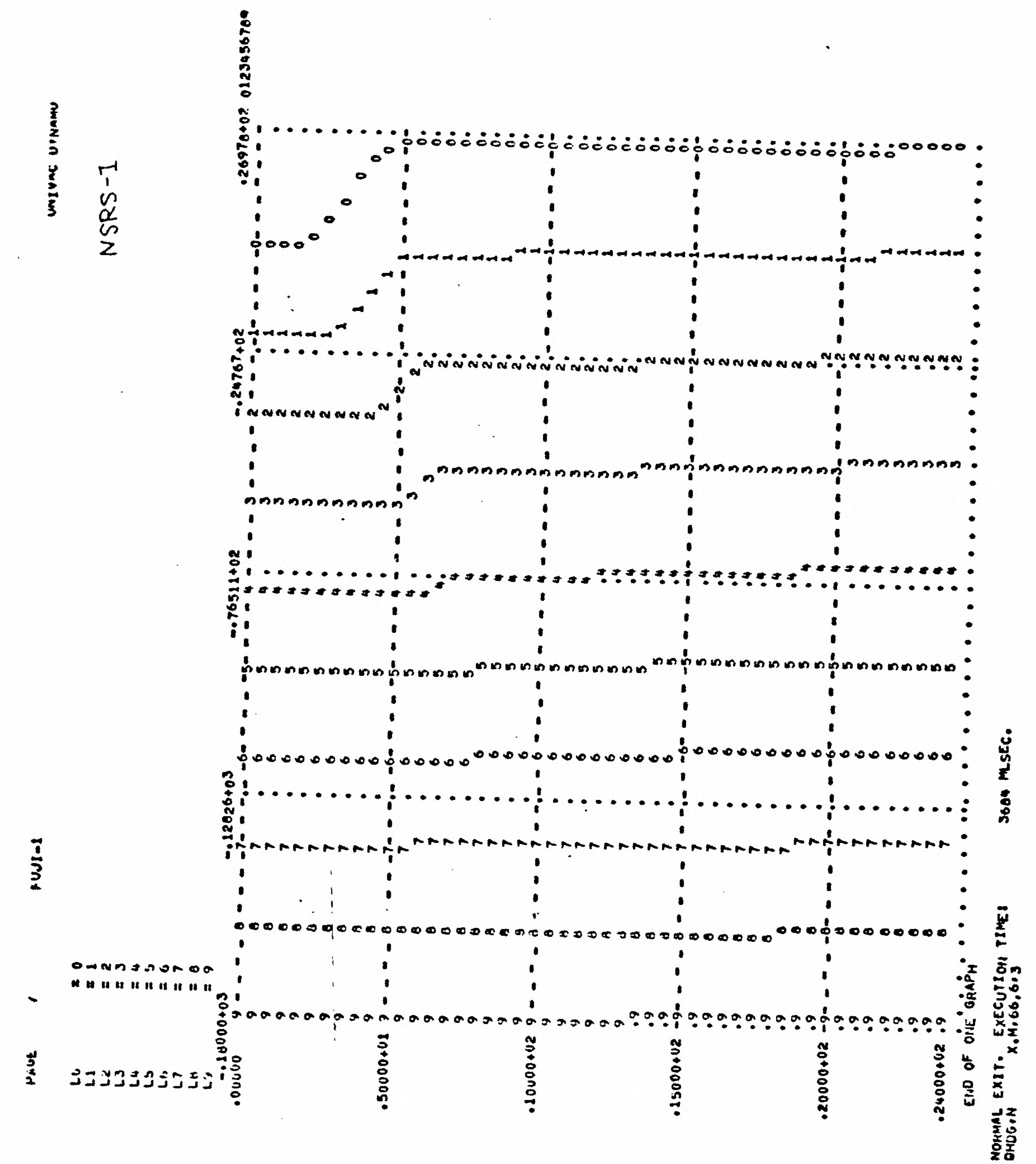
RUN ES-4

L0 = 0
L1 = 1
L2 = 2
L3 = 3
L4 = 4
L5 = 5
L6 = 6
L7 = 7
L8 = 8
L9 = 9



END OF ONE GRAPH

NORMAL EXIT. EXECUTION TIME: 3379 MLSEC.

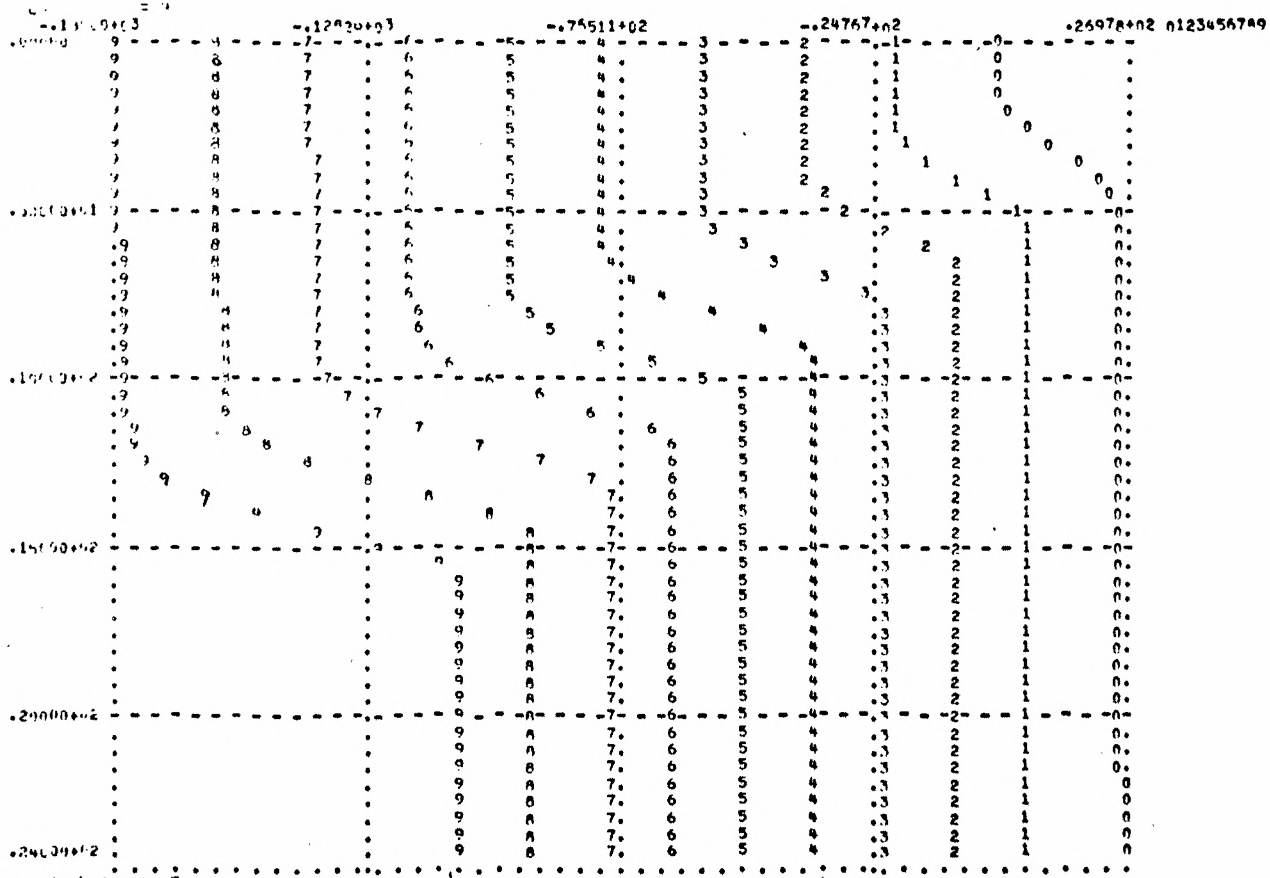

NSRS-1
FUJI-1
L0 = 0
L1 = 1
L2 = 2
L3 = 3
L4 = 4
L5 = 5
L6 = 6
L7 = 7
L8 = 8
L9 = 9
.00000
.50000+01
.10000+02
.15000+02
.20000+02
.24000+02
-.18000+03
-.12826+03
-.76511+02
-.24767+02
.26978+02 0123456789
END OF ONE GRAPH
NORMAL EXIT. EXECUTION TIME: 3684 MLSEC.

RUN NSRS-1

RUN NSRS-9

PAGE 7 FUJI-1 UNIVAC DYNAMO

NSRS-9

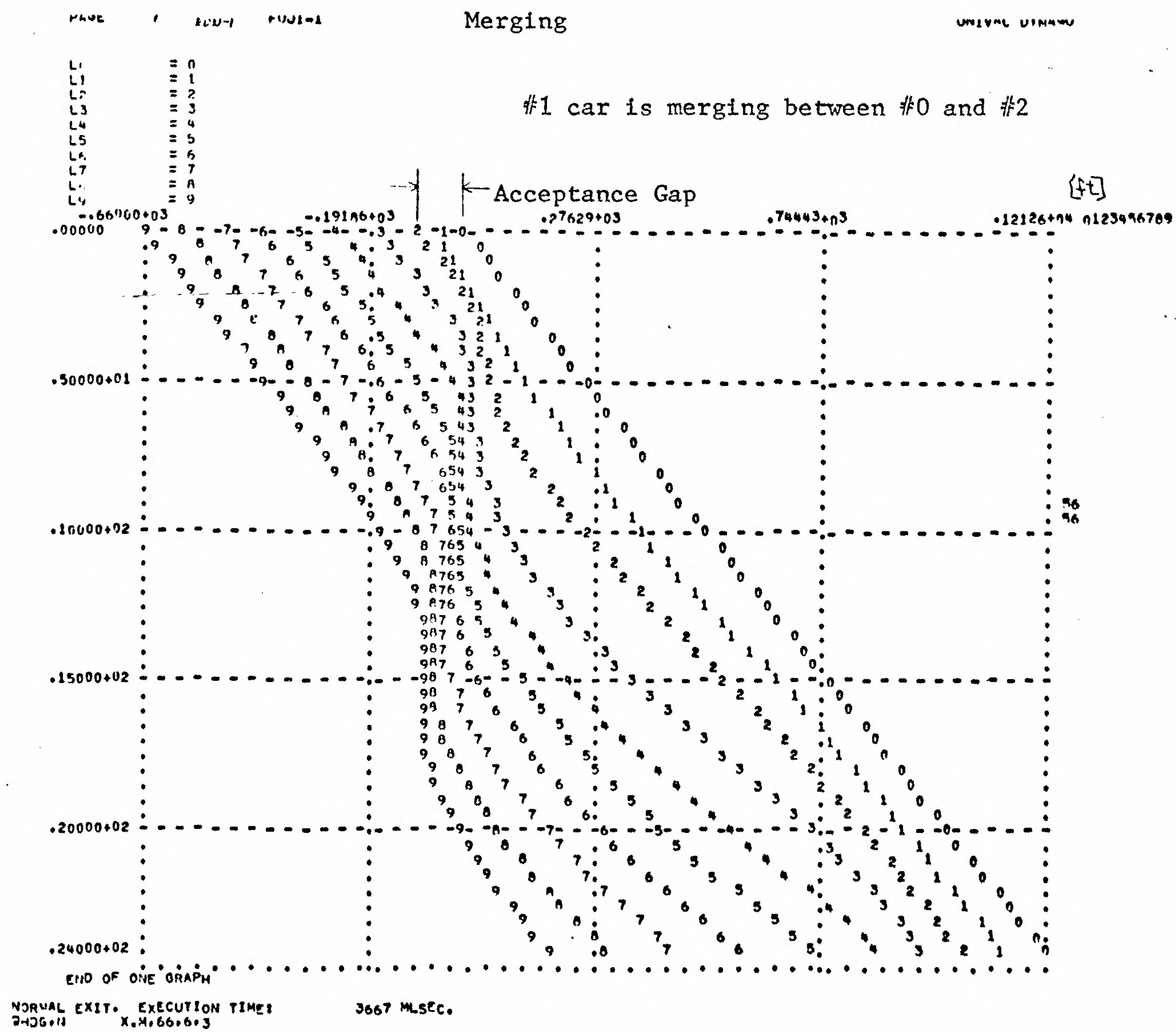


collision

END OF ONE GRAPH

NORMAL EXIT. EXECUTION TIME: 4045 MLSEC.

RUN MRG-1

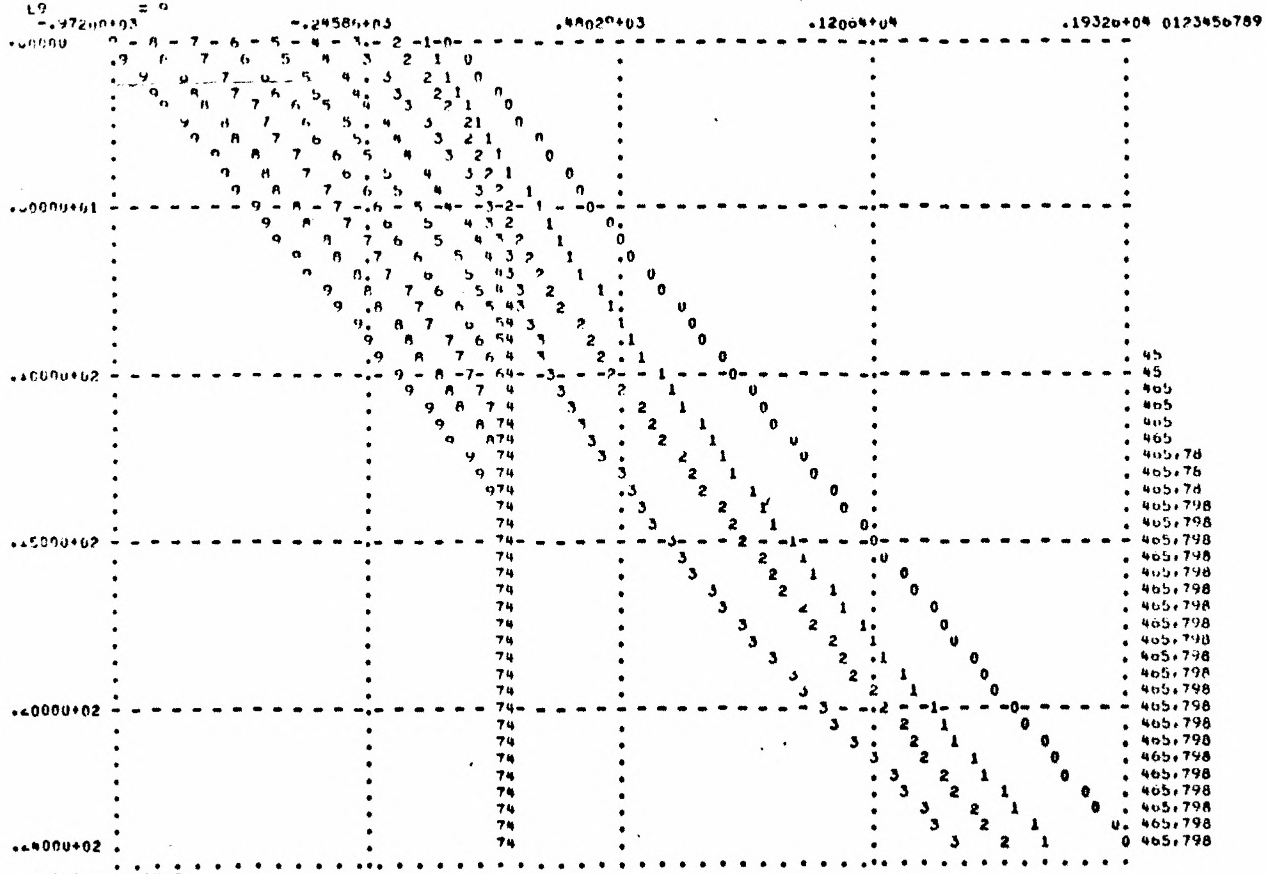


Merging
#1 car is merging between #0 and #2
Acceptance Gap
[ft]
L0 = 0
L1 = 1
L2 = 2
L3 = 3
L4 = 4
L5 = 5
L6 = 6
L7 = 7
L8 = 8
L9 = 9
-.66000+03
-.19186+03
.27629+03
.74443+03
.12126+04
0123456789
.00000
.50000+01
.10000+02
.15000+02
.20000+02
.24000+02
END OF ONE GRAPH
NORMAL EXIT. EXECUTION TIME: 3667 MLSEC.

MRG-2

RUN MRG-2

L0 = 0
L1 = 1
L2 = 2
L3 = 3
L4 = 4
L5 = 5
L6 = 6
L7 = 7
L8 = 8
L9 = 9



END OF ONE GRAPH

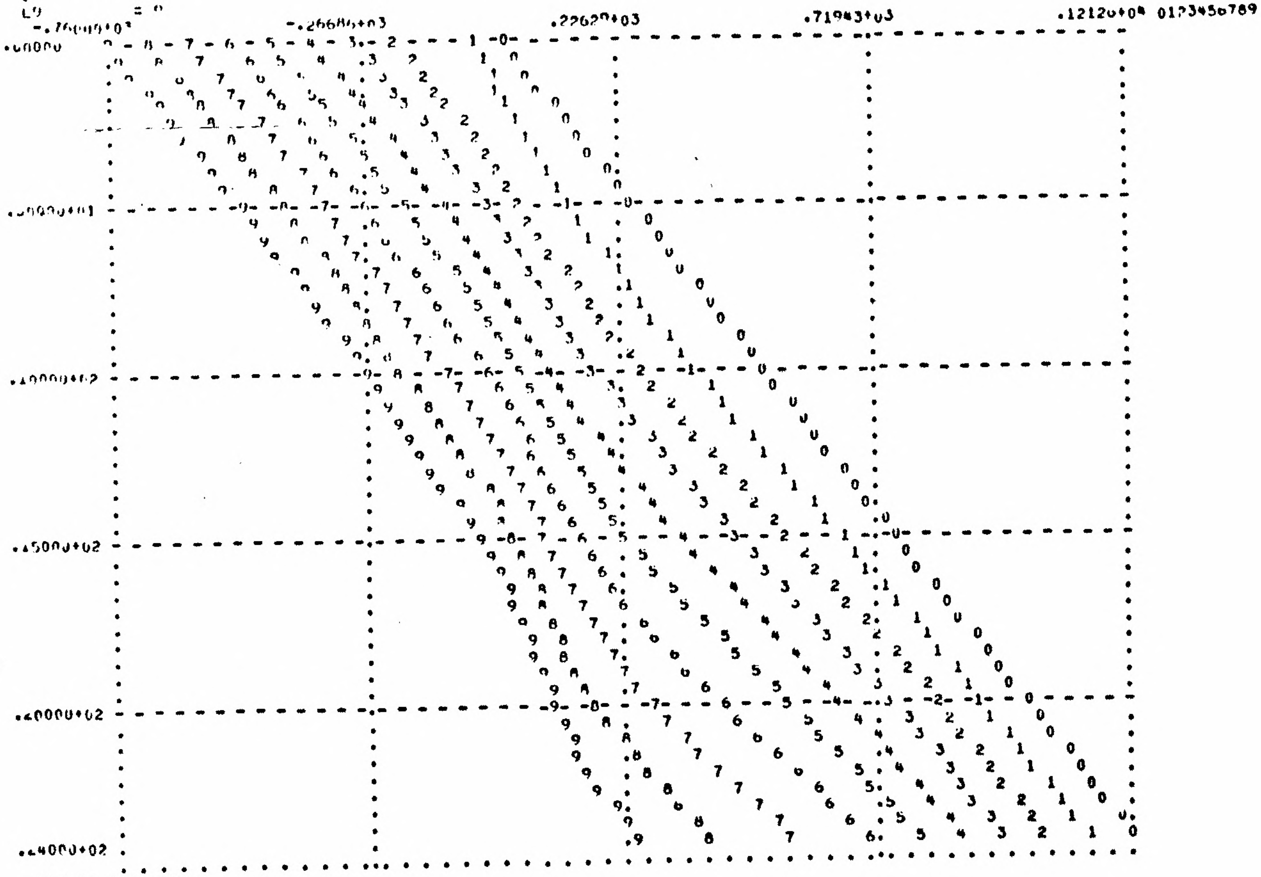
COLLISION

NORMAL EXIT. EXECUTION TIME: 3660 MLSEC.

PAGE 7 FUJI-2 UNIVAC DYNAMO

MRG-3

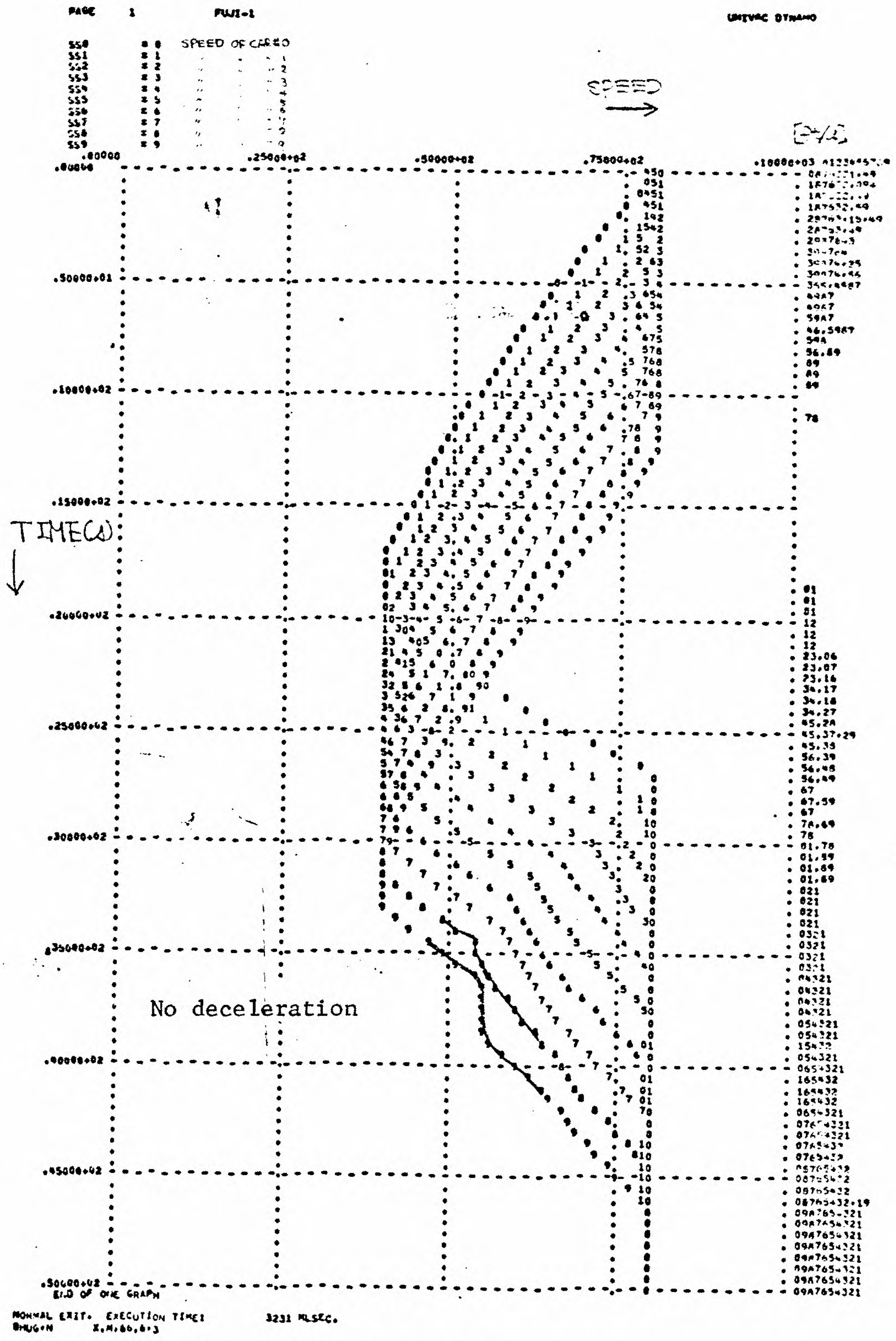
L0	=	0
L1	=	1
L2	=	2
L3	=	3
L4	=	4
L5	=	5
L6	=	6
L7	=	7
L8	=	8
L9	=	9



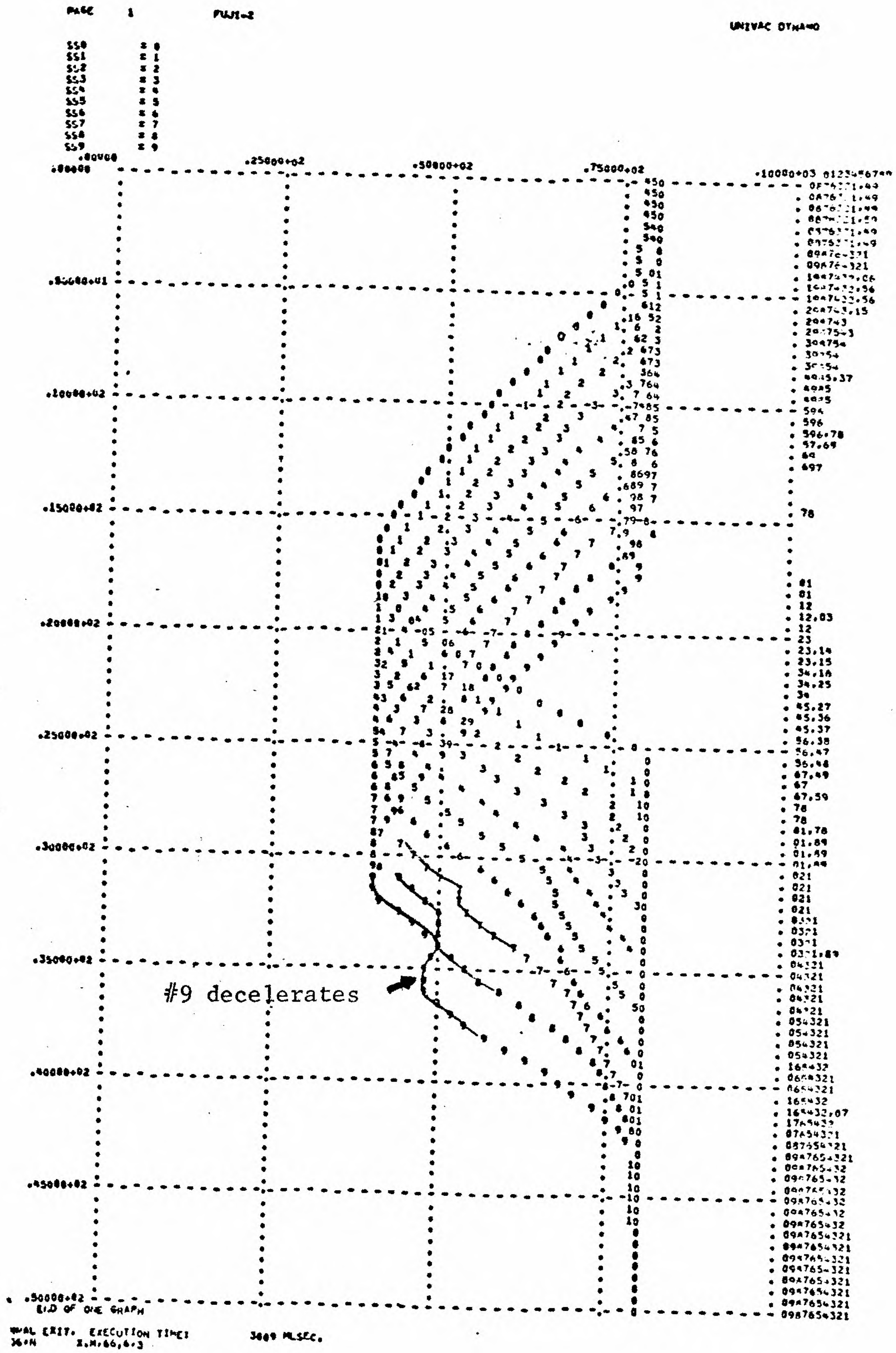
END OF ONE GRAPH

NORMAL EXIT, EXECUTION TIME: 4534 MLSEC.

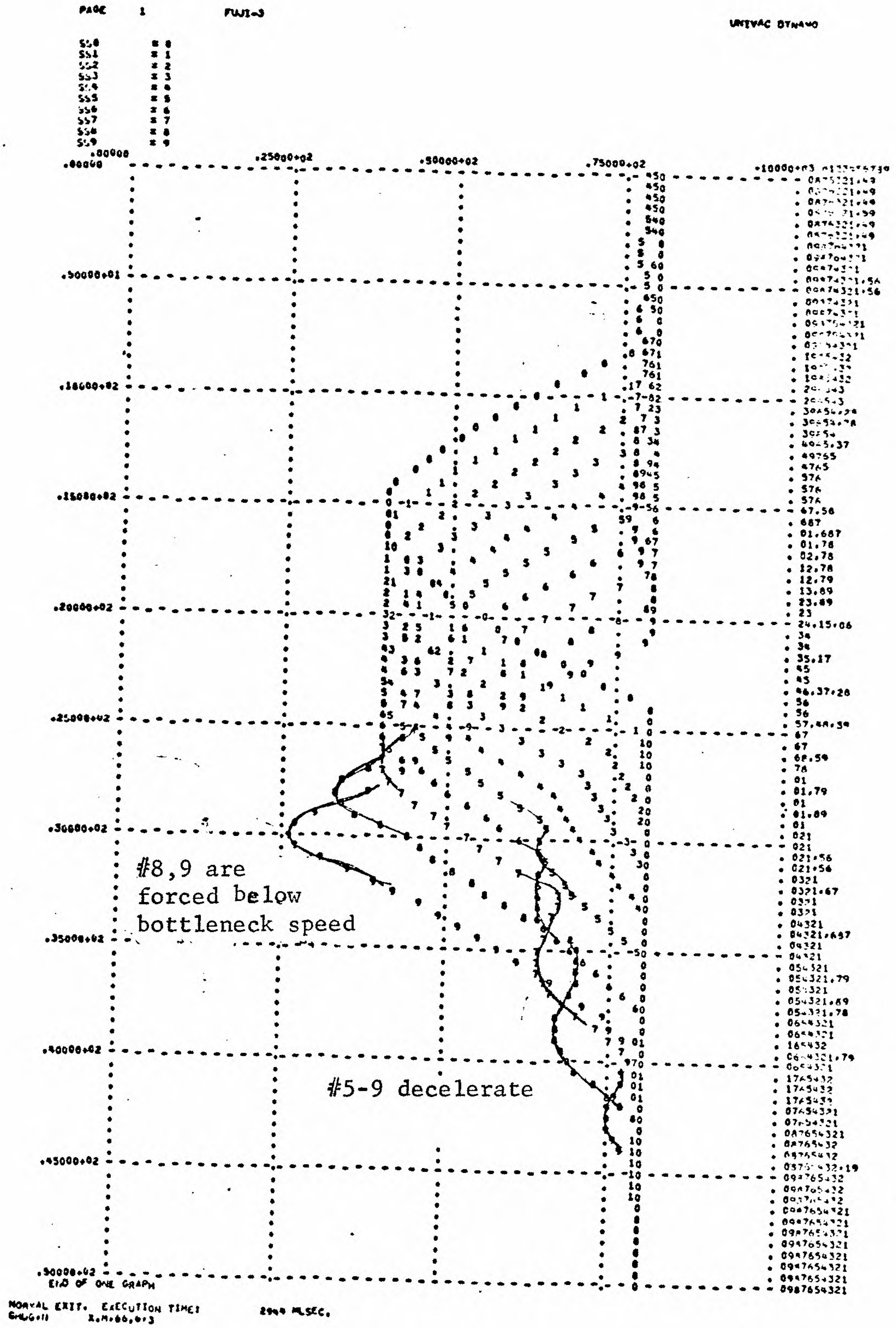
RUN MRG-3


PAGE 1
FUJI-1
UNIVAC DYNAMO
SPEED OF CAR
SPEED
TIME
No deceleration
.00000
.25000+02
.50000+02
.75000+02
.10000+03
.50000+01
.10000+02
.15000+02
.20000+02
.25000+02
.30000+02
.35000+02
.40000+02
.45000+02
.50000+02
END OF ONE GRAPH
NORMAL EXIT. EXECUTION TIME: 3231 MLSEC.

RUN BTN-1


PAGE 1
FUJI-2
UNIVAC DYNAMO
SS0 = 0
SS1 = 1
SS2 = 2
SS3 = 3
SS4 = 4
SS5 = 5
SS6 = 6
SS7 = 7
SS8 = 8
SS9 = 9
.25000+02
.50000+02
.75000+02
.10000+03
.15000+02
.20000+02
.25000+02
.30000+02
.35000+02
.40000+02
.45000+02
.50000+02
#9 decelerates
END OF ONE GRAPH
EXECUTION TIME: 3089 MLSEC.

RUN BTN-2


PAGE 1
FUJI-3
UNIVAC DYNAMO
.25000+02
.50000+02
.75000+02
.50000+01
.20000+02
.25000+02
.30000+02
.35000+02
.40000+02
.45000+02
#8,9 are forced below bottleneck speed
#5-9 decelerate
END OF ONE GRAPH
NORMAL EXIT. EXECUTION TIME:
MSEC.

RUN BTN-3

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